ME 24-221
THERMODYNAMICS I

Solutions to Assignment 10
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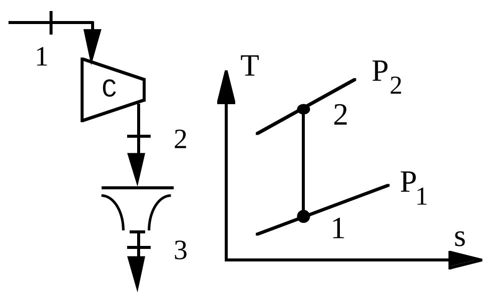
9.5 Air at 100 kPa, 17°C is compressed to 400 kPa after which it is expanded through a nozzle back to the atmosphere. The compressor and the nozzle are both reversible and adiabatic and kinetic energy in/out of the compressor can be neglected. Find the compressor work and its exit temperature and find the nozzle exit velocity.

Solution:



SSSF separate control volumes around compressor and nozzle. For ideal compressor we have inlet : 1 and exit : 2

Adiabatic : $q = 0$.

Reversible: $s_{gen} = 0$

Energy Eq.: $h_1 + 0 = w_C + h_2$;

Entropy Eq.: $s_1 + 0/T + 0 = s_2$

$$-w_C = h_2 - h_1 , \quad s_2 = s_1$$

State 1: $\Rightarrow$ Air Table A.7: $h_1 = 290.43$ kJ/kg

$$P_{r2} = P_{r1} \times P_2/P_1 = 0.9899 \times 400 / 100 = 3.98$$

State 2: $P_{r2} = 3.98$ in Table A.7 gives $T_2 = 430.5$ K, $h_2 = 432.3$ kJ/kg

$$\Rightarrow \quad -w_C = 432.3 - 290.43 = 141.86 \text{ kJ/kg}$$

The ideal nozzle then expands back down to state 1 (constant s) so energy equation gives:

$$\frac{1}{2}\mathbf{V}^2 = h_2 - h_1 = -w_C = 141860 \text{ J/kg} \quad \text{(remember conversion to J)}$$

$$\Rightarrow \quad \mathbf{V} = \sqrt{2 \times 141860} = \mathbf{532.7\ m/s}$$

9.14 A diffuser is a steady-state, steady-flow device in which a fluid flowing at high velocity is decelerated such that the pressure increases in the process. Air at 120 kPa, 30°C enters a diffuser with velocity 200 m/s and exits with a velocity of 20 m/s. Assuming the process is reversible and adiabatic what are the exit pressure and temperature of the air?

C.V. Diffuser, SSSF single inlet and exit flow, no work or heat transfer.

Energy Eq.: $h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2$, => $h_e - h_i = C_{Po}(T_e - T_i)$

Entropy Eq.: $s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e$ (Reversible, adiabatic)

Energy equation then gives:

$C_{Po}(T_e - T_i) = 1.004(T_e - 303.2) = (200^2 - 20^2)/(2\times1000)$ => $T_e = \mathbf{322.9\ K}$

$P_e = P_i(T_e/T_i)^{\frac{k}{k-1}} = 120(322.9/303.2)^{3.5} = \mathbf{149.6\ kPa}$

9.20 A certain industrial process requires a steady supply of saturated vapor steam at 200 kPa, at a rate of 0.5 kg/s. Also required is a steady supply of compressed air at 500 kPa, at a rate of 0.1 kg/s. Both are to be supplied by the process shown in Fig. P9.20. Steam is expanded in a turbine to supply the power needed to drive the air compressor, and the exhaust steam exits the turbine at the desired state. Air into the compressor is at the ambient conditions, 100 kPa, 20°C. Give the required steam inlet pressure and temperature, assuming that both the turbine and the compressor are reversible and adiabatic.

C.V. Each device. SSSF. Both adiabatic ($q = 0$), reversible ($s_{gen} = 0$)

Compressor: $s_4 = s_3$ => $T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 293.2\left(\frac{500}{100}\right)^{0.286} = 464.6\ \text{K}$

$\dot{W}_C = \dot{m}_3(h_3 - h_4) = 0.1 \times 1.004(293.2 - 464.6) = -17.2\ \text{kW}$

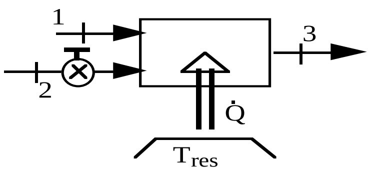
Turbine: Energy: $\dot{W}_T = +17.2\ \text{kW} = \dot{m}_1(h_1 - h_2)$; Entropy: $s_2 = s_1$

Table B.1.2: $P_2 = 200$ kPa, $x_2 = 1$ => $h_2 = 2706.6$, $s_2 = 7.1271$

$h_1 = 2706.6 + 17.2/0.5 = 2741.0$ kJ/kg

$s_1 = s_2 = 7.1271$ kJ/kg K At $h_1, s_1 \rightarrow$ $P_1 = \mathbf{242\ kPa}$, $T_1 = \mathbf{138.3°C}$

9.50 A mixing chamber receives 5 kg/min ammonia as saturated liquid at −20°C from one line and ammonia at 40°C, 250 kPa from another line through a valve. The chamber also receives 325 kJ/min energy as heat transferred from a 40°C reservoir. This should produce saturated ammonia vapor at −20°C in the exit line. What is the mass flow rate in the second line and what is the total entropy generation in the process?



CV: Mixing chamber out to reservoir

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q} = \dot{m}_3 h_3$$

$$\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{Q}/T_{res} + \dot{S}_{gen} = \dot{m}_3 s_3$$

From the energy equation:

$$\dot{m}_2 = \left[(\dot{m}_1(h_1 - h_3) + \dot{Q}\right]/(h_3 - h_2)$$

$$= \left[5 \times (89.05 - 1418.05) + 325\right]/(1418.05 - 1551.7)$$

$$= \mathbf{47.288\ kg/min} \quad \Rightarrow \quad \dot{m}_3 = 52.288 \text{ kg/min}$$

$$\dot{S}_{gen} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{Q}/T_{res}$$

$$= 52.288 \times 5.6158 - 5 \times 0.3657 - 47.288 \times 5.9599 - 325/313.15$$

$$= \mathbf{8.94\ kJ/K\ min}$$

9.80 A certain industrial process requires a steady 0.5 kg/s supply of compressed air at 500 kPa, at a maximum temperature of 30°C. This air is to be supplied by installing a compressor and aftercooler. Local ambient conditions are 100 kPa, 20°C. Using an isentropic compressor efficiency of 80%, determine the power required to drive the compressor and the rate of heat rejection in the aftercooler.

Air: $R = 0.287$ kJ/kg-K, $C_p = 1.004$ kJ/kg-K, $k = 1.4$

State 1: $T_1 = T_o = 20^oC$, $P_1 = P_o = 100$ kPa, $\dot{m} = 0.5$ kg/s

State 2: $P_2 = P_3 = 500$ kPa

State 3: $T_3 = 30^oC$, $P_3 = 500$ kPa

Assume $\eta_s = 80$ % (Any value between 70%-90% is OK)

Compressor: Assume Isentropic

$$T_{2s} = T_1 \left(P_2/P_1\right)^{\frac{k-1}{k}}, \quad T_{2s} = 464.6 \text{ K}$$

1^{st} Law: $q_c + h_1 = h_2 + w_c$; $q_c = 0$, assume constant specific heat

$$w_{cs} = C_p(T_1 - T_{2s}) = -172.0 \text{ kJ/kg}$$

$$\eta s = w_{cs}/w_c, \quad w_c = w_{cs}/\eta s = -215, \quad \dot{W}_C = \dot{m} w_C = \mathbf{-107.5\ kW}$$

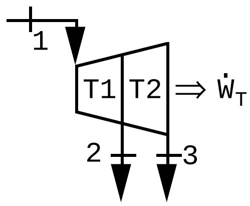
$$w_c = C_p(T_1 - T_2), \text{ solve for } T_2 = 507.5 \text{ K}$$

Aftercooler:

1^{st} Law: $q + h_2 = h_3 + w$; $w = 0$, assume constant specific heat

$$q = C_p(T_3 - T_2) = 205 \text{ kJ/kg}, \quad \dot{Q} = \dot{m} q = \mathbf{-102.5\ kW}$$

9.66 A flow of 20 kg/s steam at 10 MPa, 550°C enters a two-stage turbine. The exit of the first stage is at 2 MPa where 4 kg/s is taken out for process steam and the rest continues through the second stage, which has an exit at 50 kPa. Assume both stages have an isentropic efficiency of 85% find the total actual turbine work and the entropy generation.



C.V.: T1 Ideal

State 1: Table B.1.2, $h_1 = 3500.9$ kJ/kg, $s_1 = 6.7561$

$h_1 = h_{2s} + w_{T1,s}$; $s_1 + \emptyset = s_{2s}$

State 2s: P_2, $s_{2s} = s_1 \Rightarrow h_{2s} = 3017.9$ kJ/kg

$w_{T1,s} = 3500.9 - 3017.9 = 483$ kJ/kg

C.V. T1 Actual

$w_{T1,ac} = w_{T1,s}\eta_{T1} = 410.5 = h_1 - h_{2ac} \Rightarrow h_{2ac} = 3090.4$ kJ/kg

State 2ac: P_2 $h_{2,ac} \Rightarrow s_{2ac} = 6.8802$ kJ/kg K

C.V. T2 Ideal

$h_{2ac} = h_{3,s} + w_{T2s}$; $s_{2ac} + \emptyset = s_{3s}$

State 3s: P_3 , $s_{3s} = s_{2ac} \Rightarrow x_{3s} = (6.8802-1.091)/6.5029 = 0.890$,

$h_{3s} = 340.5 + 0.89\times2305.4 = 2392.9$ kJ/kg

$w_{T2s} = 3090.4 - 2392.9 = 697.5$ kJ/kg

C.V. T2 Actual

$w_{T2,ac} = w_{T2s}\eta_{T2} = 592.9 = h_{2ac} - h_{3ac}$ => $h_{3ac} = 2497.5$

State 3ac: P_3 , $h_{3ac} \Rightarrow x_{3ac} = (2497.5-340.5)/2305.4 = 0.9356$,

$s_{3ac} = 1.091 + 0.9356\times6.5029 = 7.1754$

C.V. T1 + T2 Actual

$\dot{W}_T = \dot{m}_1 w_{T1ac} + (\dot{m}_1 - \dot{m}_2) w_{T2ac} = 20 \times 410.5 + 16 \times 592.9 =$ **17696 kW**

$\dot{S}_{gen} = \dot{m}_2 s_{2ac} + \dot{m}_3 s_{3ac} - \dot{m}_1 s_1 = 4\times6.8802 + 16\times7.1754 - 20\times6.7561$

= **7.20 kW/K**