Solution to Problem 1:

There are two correct answers. Either is acceptable
Answer 1: The edges may be inserted in the order $(1, 2)$, $(2, 3)$, $(4, 5)$, $(2, 4)$.
Answer 2: $(2, 3)$, $(1, 2)$, $(4, 5)$, $(2, 4)$.

Solution to Problem 2:

Answer 1: $(5, 4)$, $(4, 2)$, $(2, 3)$, $(1, 2)$,
or
Answer 2: $(5, 4)$, $(4, 2)$, $(1, 2)$, $(2, 3)$.

Solution to problem 3:

Each line represents the t_v after each iteration, and which time the vertex enters S:

1. $t_1 = \infty$, $t_2 = 15$, $t_3 = 12$, $t_4 = 8$. Vertex 4 enters S.

2. $t_1 = 38$, $t_2 = 15$, $t_3 = 12$, $t_4 = 8$. Vertex 3 enters S.

3. $t_1 = 26$, $t_2 = 15$, $t_3 = 12$, $t_4 = 8$. Vertex 2 enters S.

4. $t_1 = 22$, $t_2 = 15$, $t_3 = 12$, $t_4 = 8$. Vertex 1 enters S.

Solution to problem 4

We proceed by induction. Suppose that we have the best distances to a given set S of vertices. Dijkstra's algorithm then takes, for each vertex in S,

each edge leading outside of S to get a new estimate for any vertex v adjacent to something in S. The cheapest way to get to any vertex outside S must be the one minimizing $t_u + c_{uv}$, over $u \in S$ and $v \in V \setminus S$. Therefore, when this v gets entered, we must also have the correct estimate for v. This last step is where the fact that all edges are nonnegative are crucial.

Solutiomn to problem 5

Any correct solution must have an edge of negative cost. They should trace the algorithm and demonstrate that their example gives wrong answers when Dijkstra's algorithm applies to get full credit. 3 points for a vague explanation.

Solution to problem 6

Let M be a matching saturating all vertices in S. If M is maximum then we are done. If not, then there is an M-augmenting path. An M-augmenting path is one that has an odd number of edges, so that the first and last vertex on the path are not saturated. Thus, the "even"-positioned edges on the path are in the matching. If we instead switch to the "odd"-positioned edges on the path, we get a larger matching M^* that now matches a set of all edges on the path, without affecting any others. Therefore, the set of vertices saturated by M^* contains the set of vertices saturated by M, so S is still saturated by the larger matching M^*. We continue this procedure.

Solution to problem 7

Let M be a maximal matching, and S be the set of all vertices saturated by M. Thus S is the set of ends of all edges in M. As we have seen, any cover is as big as M, and S is twice as big as M. Therefore, S is at most twice the size

of a minimum cover.

Since M is a maximal matching, there cannot be an edge, neither end of which is in S – else we could add that edge to get a matching strictly containing M. Therefore, S must be a vertex cover.