

1. Consider the hashing example in class. Again, we pick a random m -element subset S of $U = \{0, \dots, N-1\}$, and as discussed in class, we hash them into $H = \{0, \dots, n-1\}$. Suppose that we pick u at random from S (therefore, we know that u is in the table). What is the expected number of comparisons needed to find u ?

2. Suppose that a box is filled with n distinguishable coupons. We draw a coupon at random, and then replace it. Let X be the number of tries needed for all coupons to have been drawn at least once. What is $E(X)$?

3. Look at section 1.1.2 of the notes for this week's lectures. Prove that:

$$\sum_{k=0}^{n-1} E(T_1 | f(u) = k) = \sum_{k=0}^{n-1} E\left(\frac{X_k}{m} \frac{1 + X_k}{2} + \left(1 - \frac{X_k}{M}\right) X_k\right)$$