21-228 Homework 6

Due October 31, 2001, in lecture

1. Suppose a gambler enters a casino with 1 dollar, and plays a fair game. Each time he plays the game, he wins with probability 1/2 and loses with probability 1/2. Each time he plays, \$1 is at stake. Suppose that the gambler picks a goal of \$$n$ that he would like to leave with. Find (and prove) a formula for the probability that he will successfully reach his goal of \$$n$ before going broke. Hint: Let $f(k, n)$ be his probability of leaving with n dollars if he enters with k dollars. Your objective is to find $f(1, n)$ – what is $f(k_0, n)$ in terms of other $f(k, n)$?

2. Let Ω be a probability space, and $B_1, \ldots, B_n$ partition the space (i.e. the B_i are pairwise disjoint events whose union is Ω). Let A be any event in the space. Prove:

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

3. Suppose we flip n loaded coins, where each coin is heads with probability p. Then $\Omega = H, T^n$. If $A \subset [n]$, let Δ_A be the set of outcomes where all coins in A come up heads. Show that if $A \cap B = \emptyset$, then Δ_A and Δ_B are independent.

4. Suppose you run an experiment. Each time you run it, there is a p probability of success, a q probability of failure, and a r probability of a "neutral" outcome. Suppose you run the experiment until you either fail or succeed. What is the probability that the last trial will be a success?

5. Consider the game of craps. You roll a pair of dice. If you roll a 2, 3, or 12, you lose. If you roll a 7 or 11, you win. If you roll someting else, the number x that you rolled is called your "point".

If you have a "point", your objective is then to roll x again before you roll a 7. You keep rolling this pair of dice until you either get another x or a 7.

You win if you roll x before getting a 7, and you lose otherwise. What is the probability that you will win? Hint: problem 4 will come in very handy.

6. Suppose now there are three dice. Before the dice are rolled, you pick a certain number x from 1 to 6. For each x that is showing after the dice are rolled, you win one dollar. If no x shows you lose a dollar. Is this a good bet for you to make? Justify your answer.

7. I have three shells. I randomly place a ball under one of these three shells. I know where the ball is, but you do not. You pick a shell, and if the shell you pick covers the ball you win. Let's say you pick a shell. Then I remove a shell (other than the one that you picked) which I know does not have the ball under it. Then there are two shells remaining, and your ball must be under one of those two, one of which is the shell you picked. What is your probability of winning?