

## 21-228 Homework 5

Due October 17, 2001

1. Use generating functions to solve  $a_{n+3} = 3a_{n+2} - 3a_{n+1} + a_n$  where  $a_0 = a_1 = a_2 = 1$ .

2. Solve the following variant of Fibonacci's problem. each mature pair of rabbits present at the end of any given month produces three more pairs of rabbits during the *next* month. Further, the baby rabbits take a month to mature. If we start with 10 baby rabbits (so no procreation takes place until the second month), how many rabbits are present after  $n$  months?

3. Suppose that we have the recurrence relation  $a_{n+2} - 2ra_{n+1} + r^2a_n = 0$ , for some real nonzero  $r$ . Find a general solution for the relation in terms of  $a_0$  and  $a_1$ .

4. Solve the recurrence relation  $a_{k+1} = a_k + 2^k$  with  $a_0 = 2$ .

5. Find the generating function for the solution to  $a_{n+2} = 3a_{n+1} + 2^n$  for  $a_0 = 1$ .