

21-228 Homework 1

Due Tuesday, September 11

All answers must be justified to receive credit.

1. How many ways can you distribute three different pieces of candy to five children if no child can receive more than one piece? What about if any child can receive any number?

2. A coffee machine allows you to have your coffee either plain, or with single or double portions of coffee and/or cream. In how many ways can you choose your coffee?

3. For any two sets A and B , we define the set $A - B$ to be the set of all elements of A that are not in B . Suppose now that every element of Y is also an element of X . Use the sum principle to prove that $|X - Y| = |X| - |Y|$.

4. Using induction, prove the generalized product principle.

5. Let A and B be sets where A has n elements and B has k elements. Here, give only brief explanations, not detailed proofs.

(a) Explain why the number of functions from A to B is the same as the number of ways of listing n elements from a set of k elements, with repetition allowed.

(b) Explain why the number of *one-to-one* functions from A to B is the same as the number of ways of listing n elements from a set of k elements, *without* repetition allowed.

6. Use the extended pigeonhole principle to show that for any set of seven integers at least three of the positive differences between them have the same last digit.