Handout #2:
An Application of Related Rates to Business

Problem: The wholesale price of apples is related to the daily supply (in thousands of crates) by the equation

$$pq - 10p - 2q + 172 = 0,$$

where p is the price (in dollars) per crate, and q is the number of crates (in thousands) supplied per day. If there are currently 6000 crates available and the supply is decreasing at the rate of 200 crates per day, at what rate is the price changing?

Solution:

Step 1 Read the problem carefully...done

Step 2 Sketch a picture is possible...not very helpful for this problem

Step 3 Identify all relevant variables and quantities

q is the number of crates available (in thousands)

p is the current price per crate (in dollars)

$q = 6$ is the current number of crates available (in thousands)

$\dfrac{dq}{dt} = -0.2$ is the rate at which the number of crates is decreasing (in thousands)

$\dfrac{dp}{dt} =$??? we want to find this rate

Step 4 Obtain equations relating the variables: for this we are given that

$$pq - 10p - 2q + 172 = 0.$$

Step 5 Differentiate to find the desired derivative $\frac{dp}{dt}$ (remember that we are thinking of p and q as functions of time t, so we need to use the chain rule):

first we differentiate both sides

$$\frac{d}{dt}[pq - 10p - 2q + 172] = \frac{d}{dt}[0]$$

$$\frac{d}{dt}[pq] - 10\frac{d}{dt}[p] - 2\frac{d}{dt}[q] + \frac{d}{dt}[172] = 0$$

$$q\frac{d}{dt}[p] + p\frac{d}{dt}[q] - 10\frac{dp}{dt} - 2\frac{dq}{dt} = 0$$

$$q\frac{dp}{dt} + p\frac{dq}{dt} - 10\frac{dp}{dt} - 2\frac{dq}{dt} = 0$$

next we collect terms involving $\frac{dp}{dt}$ to one side and everything else to the other side

$$q\frac{dp}{dt} - 10\frac{dp}{dt} = 2\frac{dq}{dt} - p\frac{dq}{dt}$$

factor out $\frac{dp}{dt}$

$$\frac{dp}{dt}\{q - 10\} = 2\frac{dq}{dt} - p\frac{dq}{dt}$$

and now solve for $\frac{dp}{dt}$

$$\frac{dp}{dt} = \frac{2\frac{dq}{dt} - p\frac{dq}{dt}}{q - 10}$$

Step 6 Plug in values. The only thing we still need in order to do this is the value of p; using our formula, we can find it:

$$pq - 10p - 2q + 172 = 0 \implies p(6) - 10p - 2(6) + 172 = 0$$
$$\implies 4p = 160$$
$$\implies p = 40.$$

Now plugging in the values for q, p and $\frac{dq}{dt}$ into our formula for $\frac{dp}{dt}$ (from Step 5), we find that

$$\frac{dp}{dt} = \frac{2\frac{dq}{dt} - p\frac{dq}{dt}}{q - 10} = \frac{2(-0.2) - 40(-0.2)}{6 - 10}$$
$$= \frac{7.6}{-4} = -1.9$$

So $\frac{dp}{dt} = -1.9$, which means that the price per crate is decreasing by $1.90 per day.