

Concepts of Math: Recitation 5

September 11, 2015

Sound and Image Check

Ask your students to flag you when they do not hear you well or do not understand what is written on the board. Most blackboards are too small. You probably erase a lot. Ask your students whether they wrote everything down before erasing the board.

Sets

1. Find the power set of the following sets:
 - (a) $A = \{\emptyset, \{\emptyset\}\}$.
 - (b) $B = \{x, y, z\}$.
2. Determine which of the following statements are true in the case of three arbitrary sets P , Q , and R . Explain.
 - (a) If P is an element of Q and if Q is a subset of R , then P is an element of R .
 - (b) If P is an element of Q and if Q is a subset of R , then P is also a subset of R .
 - (c) If P is a subset of Q and Q is an element of R , then P is an element of R .
 - (d) If P is a subset of Q and Q is an element of R , then P is a subset of R .
3. Prove the following identity for arbitrary sets A , B and C using a double-containment argument.

$$A \cap (B - C) = (A \cap B) - (A \cap C).$$

4. Let A and B be two sets. Prove that $A \cup B = B$ if and only if $A \subseteq B$.
5. For every $n \in \mathbb{N}$, let $A_n = \{m \mid m \in \mathbb{Z}, n \leq m < 2n\}$. Find the following sets.

(a)

$$\bigcup_{n=2}^4 A_n$$

(b)

$$\bigcup_{n=1}^{\infty} A_n$$

(c)

$$\bigcap_{n=1}^{\infty} A_n$$

6. True or false? In each case, provide a proof or a counterexample.

(a) If $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$.

(b) If $A \times B \subseteq C \times D$, then $A \subseteq C$ and $B \subseteq D$.

(c) $A \subseteq C$ and $B \subseteq D$ if and only if $A \times B \subseteq C \times D$.

(d) If $A \cup B \subseteq A \cap B$, then $A = B$.