Concepts of Math: Recitation 4

September 9, 2015

Quantifiers

1. Find the truth values of the following statements. Explain your answers.
   (a) There exists an integer x such that $x^3 + 1 = 0$.
   (b) For all real numbers x and y, $x^2 + y^3 \geq 0$.
2. Find all the real numbers a such that the following statement is true. "For all $x \in \mathbb{R}$, there is a unique y such that $x^4 y + ay + x = 0$."
3. Find the truth value of the following statement and explain your answer. "If $n \in \mathbb{N}$ and $n^2 + (n+1)^2 = (n+2)^2$, then $n = 3$."
4. Negate the following statement about the sequence $a_1, a_2, a_3, \ldots$.

   For every $\epsilon > 0$, there exists a positive integer N, such that, for all $n \geq N$, $|a_n - L| < \epsilon$.
5. (a) Show that $\forall x P(x) \vee \forall x Q(x)$ is equivalent to $\forall x \forall y (P(x) \vee Q(y))$
   (b) Show that $\forall x P(x) \wedge \exists x Q(x)$ is equivalent to $\forall x \exists y (P(x) \wedge Q(y))$
6. Rewrite each of these statements so that negations appear only within predicates.
   (a) $\neg \forall x \exists y P(x, y)$
   (b) $\neg \forall x \forall y (P(x, y) \vee Q(x, y)$
   (c) $\neg(\exists x \exists y \neg P(x, y) \wedge \forall x \forall y Q(x, y))$
   (d) $\neg \forall x (\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$

Introduction to Sets

1. Define the cardinality of a set as the number of elements in it. What is the cardinality of each of these sets?
   (a) $\emptyset$
   (b) $\{\emptyset, \{a\}\}$
   (c) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

2. In class I explained that $a \in A$ means that a is an element of A. Please define subsets ($A \subseteq B$) and equality of sets ($A = B$). Find two sets A and B such that $A \in B$ and $A \subseteq B$.

3. Decide whether the satements below are true or false.

   (a) $1 \in \{\{1\}, 2\}$

   (b) $\{1\} \in \{\{1\}, 2\}$

   (c) $\{1\} \subseteq \{\{1\}, 2\}$

   (d) $\{\{1\}\} \in \{\{1\}, 2\}$