Concepts of Math: Recitation 21

November 9, 2015

Linear Diophantine Equations

In lecture we discussed linear Diophantine equations, that is equations $ax + by = c$, where a, b, c are given integers and x, y are the integer unknowns. We showed that this equation has a solution if and only if c is a multiple of $\gcd(a, b)$.

1. Give 2-3 examples of linear Diophantine equations that have or don't have solutions. Do not solve them.

2. Consider the function $f : \mathbb{Z}^2 \to \mathbb{Z}$ defined by
$$f(x, y) = 45x + 25y.$$
Is f injective? Is f surjective? Prove your answers. Find the range of f.

3. Let $\gcd(a, b) = 1$. Prove that the set of all the integer solutions of equations $ax+by = 0$ is $(x, y) = (bk, -ak)$ where $k \in \mathbb{Z}$. Use the following fact we proved in class: if $\gcd(a, b) = 1$ and $a \mid bq$, where $q \in \mathbb{Z}$, then $a \mid q$.

4. Let $a, b \in \mathbb{Z}$, not both zero. Prove that the set of all the integer solutions of equations $ax + by = 0$ is
$$(x, y) = (\frac{bk}{\gcd(a, b)}, -\frac{ak}{\gcd(a, b)})$$
where $k \in \mathbb{Z}$.

5. Find all the solutions to the linear Diophantine equation $100x + 148y = 0$.

The Euclidean Algorithm

1. Use the Euclidean Algorithm to find $\gcd(112, 36)$.

2. Use back substitution in the previous problem to find a pair of integers m, n such that $4 = 112m + 36n$. You found a particular solution of the linear Diophantine equation $112x+36y = 4$. Find a particular solution of the linear Diophantine equation $112x + 36y = 12$.

3. Explain how this method from the previous problem will produce a particular solution of the linear Diophantine equation $ax + by = c$, where $gcd(a, b) \mid c$.