Concepts of Math: Recitation 2

August 31, 2015

Implication Game

Make up 3 – 4 implications. Do not use quantifiers in you statements, we haven's done them yet. For each implication $p \to q$ ask the students to do the following.

1. Formulate the converse implication $q \to p$. Note that its meaning is different from the original.
2. Formulate the contrapositive $\neg q \to \neg p$. Note that its meaning is the same as the original.
3. Reformulate the original implication as $\neg p \vee q$. Note that its meaning is the same as the original.

Here is an example: "If it rains, then it pours." Converse: "If it pours, then it rains." Contrapositive: "If it does not pour, then it does not rain." The $\neg p \vee q$ form: "It does not rain or it pours."

Negation Game

Remind the students De Morgan's Laws of Logic $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ and $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$. Also remind them that $p \to q \Leftrightarrow \neg p \vee q$. It follows that

$$\neg(p \to q) \Leftrightarrow \neg(\neg p \vee q) \Leftrightarrow p \wedge \neg q.$$

Thus the negation of an implication is not an implication. For example the negation of "If it rains, then it pours" is "It does rain and it does not pour". Please come up with 4 – 6 "and" and "or" statements for the students to negate using De Morgan's Laws of Logic, like "I love bananas and I do not love oranges." Also about 4 – 6 implications for students to negate. Do not use quantifiers in you statements, we haven's done them yet.

Logical Equivalences

1. Simplify the following statements.

   (a) $(p \wedge q) \vee (\neg((\neg p) \vee q))$

   (b) $(p \vee q) \wedge ((\neg p) \rightarrow (\neg q))$

   (c) $(p \vee r) \rightarrow ((q \vee (\neg r)) \rightarrow ((\neg p) \rightarrow r))$

2. Write this definition on the board. A compound statement on p_1, p_2, ..., p_n is said to be in disjunctive normal form (DNF) if it looks like

$$(a_{11} \wedge a_{12} \ldots \wedge a_{1n}) \vee (a_{21} \wedge a_{22} \ldots \wedge a_{2n}) \vee \ldots \vee (a_{m1} \wedge a_{m2} \ldots \wedge a_{mn})$$

   where, for each i and j, $1 \leq i \leq m$, $1 \leq j \leq n$, either $a_{ij} = p_j$ or $a_{ij} = \neg p_j$ and all minterms $a_{i1} \wedge a_{i2} ... \wedge a_{in}$ are distinct. Give examples of statements that are DNF and statements that are not DNF.

3. Show that $x \Leftrightarrow ((x \wedge y) \vee (x \wedge (\neg y)))$. Use this property to write the DNF for $p \vee q$.