Department of Mathematical Sciences   Spring 2002
Carnegie Mellon University

21-112 Calculus II

Optimization with Constraints:
outline of solution method

Statement of problem:

Minimize (or maximize): $f(x, y)$

Subject to: $g(x, y) = 0$

Step 1: Find all critical points for g such that $g(x, y) = 0$. In other words, find all values for x and y that simultaneously satsify

$$g_x(x, y) = 0;$$
$$g_y(x, y) = 0;$$
$$g(x, y) = 0.$$

Step 2: Find all critical points for $L = f + \lambda g$ such that $g(x, y) = 0$. In other words, find all values for x, y and λ that simultaneously satisfy

$$f_x(x, y) = -\lambda g_x(x, y);$$
$$f_y(x, y) = -\lambda g_y(x, y);$$
$$g(x, y) = 0.$$

Step 3:   – If only one point is found from steps 1 and 2,
then assume this is the solution.
– If more then one point is found from steps 1 and 2,
then plug these points into f and compare the resulting values:
   * If all points yield the same value for f, then assume the points found are all solutions.
   * Otherwise, the largest value obtained is the maximum for f and the smallest value obtained for f is the minimum.