15-451 Mini 2

Jan 28, 2008

This mini is due via *email* to your TA, by midnight Tuesday Feb 5.
Please use the subject line "15-451 MINI #1" in your email.
Questions/concerns/comments to Dafna Shahaf ⟨dshahaf+451@cs.cmu.edu⟩

1 Question 1: Median Finding

In class, we discussed a deterministic linear-time algorithm for finding the median (or kth smallest element) of an unsorted array. Our analysis of this algorithm gave the recurrence:

$$T(n) \leq T(n/5) + T(7n/10) + cn$$

Which can be shown to be $O(n)$. Suppose we changed the algorithm so that rather than breaking up the array into groups of size 5, we used groups of size 3 instead. Show that the new algorithm is no longer $O(n)$.

2 Question 2: Sorting

1. Show that it is possible to sort any array of 4 elements using only 5 comparisons. Note: There are multiple correct ways to do this.
2. Is it possible to sort every array of size 4 using only 4 comparisons? Why or why not?

3 Question 3: Balls and Bins

You throw m balls into n bins independently. Each bin is equally likely to receive each ball.

1. What is the expected number of pairs which collide (end up in the same bin)?
2. How large does m have to be (in terms of n) for this expectation to exceed one?
3. What is the probability (in terms of n, m) that there are no collisions?
4. How is this related to the Birthday Paradox?