Uninformed Search

Day 1 & 2 of Search

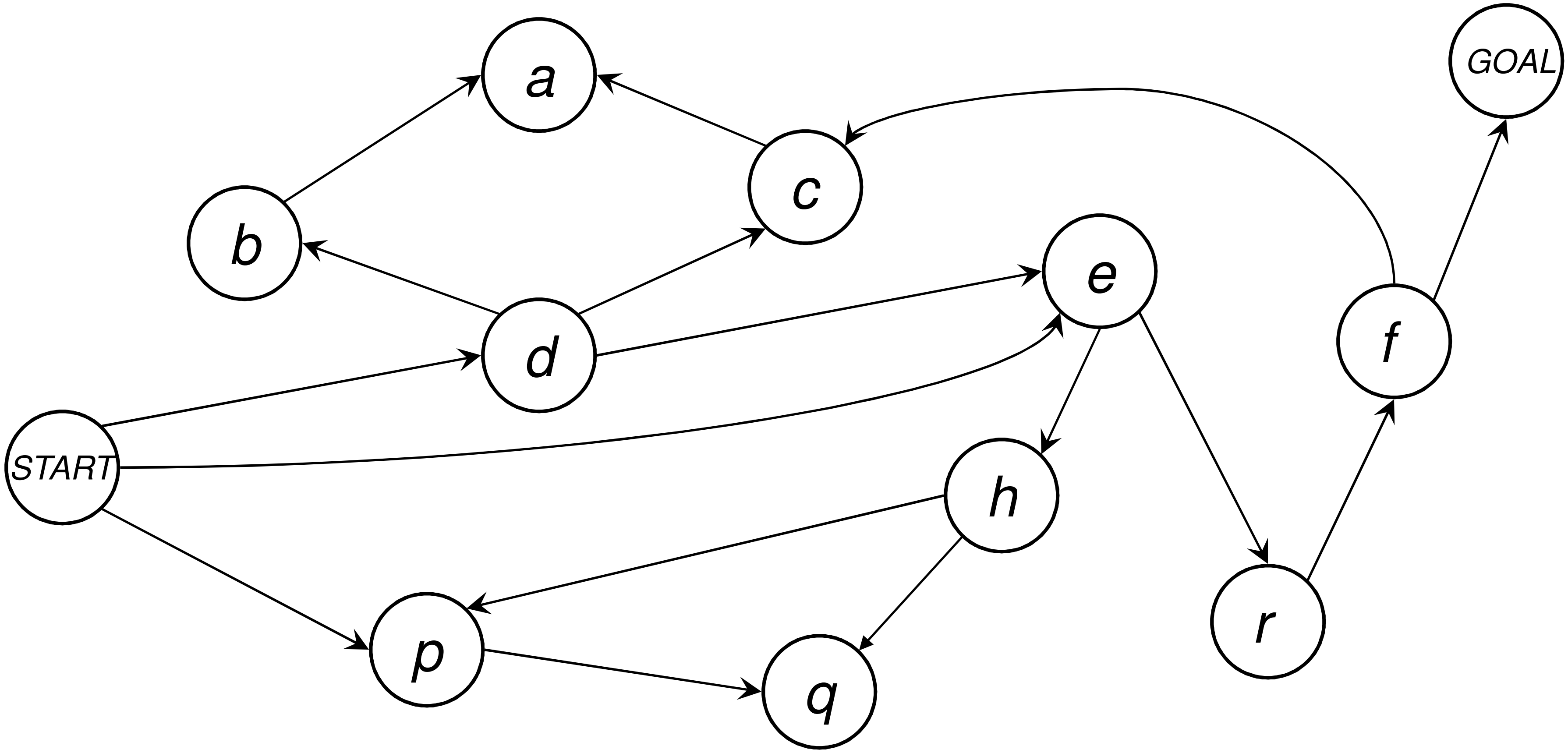
Russel & Norvig Chap. 3

Material in part from http://www.cs.cmu.edu/~awm/tutorials

Search

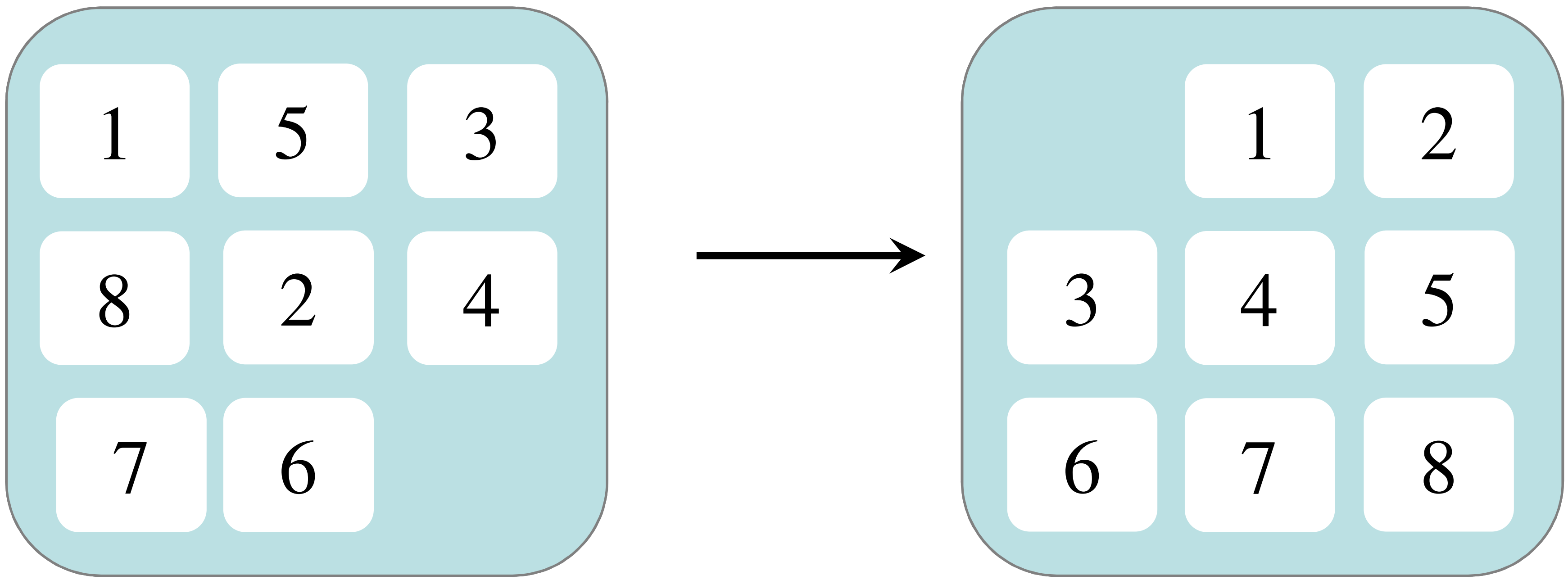
- Examples of Search problems?
- The Oak Tree
- Informed versus Uninformed
  - Heuristic versus Blind

A Search Problem



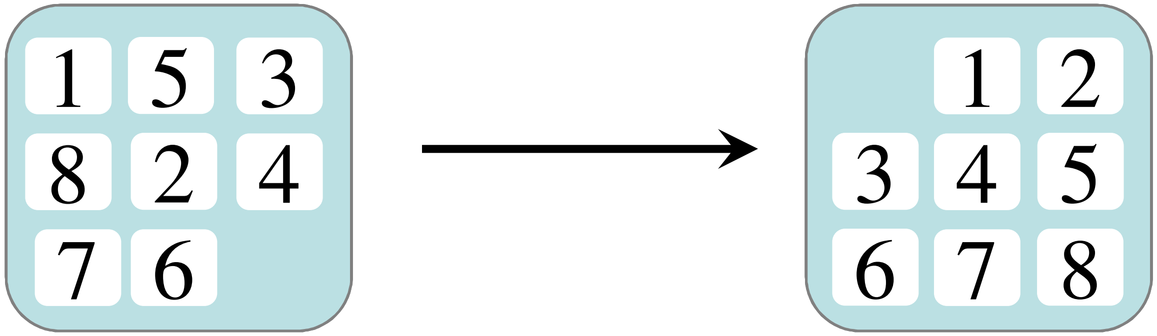
- Find a path from START to GOAL
- Find the minimum number of transitions

Example



START *GOAL*

Example

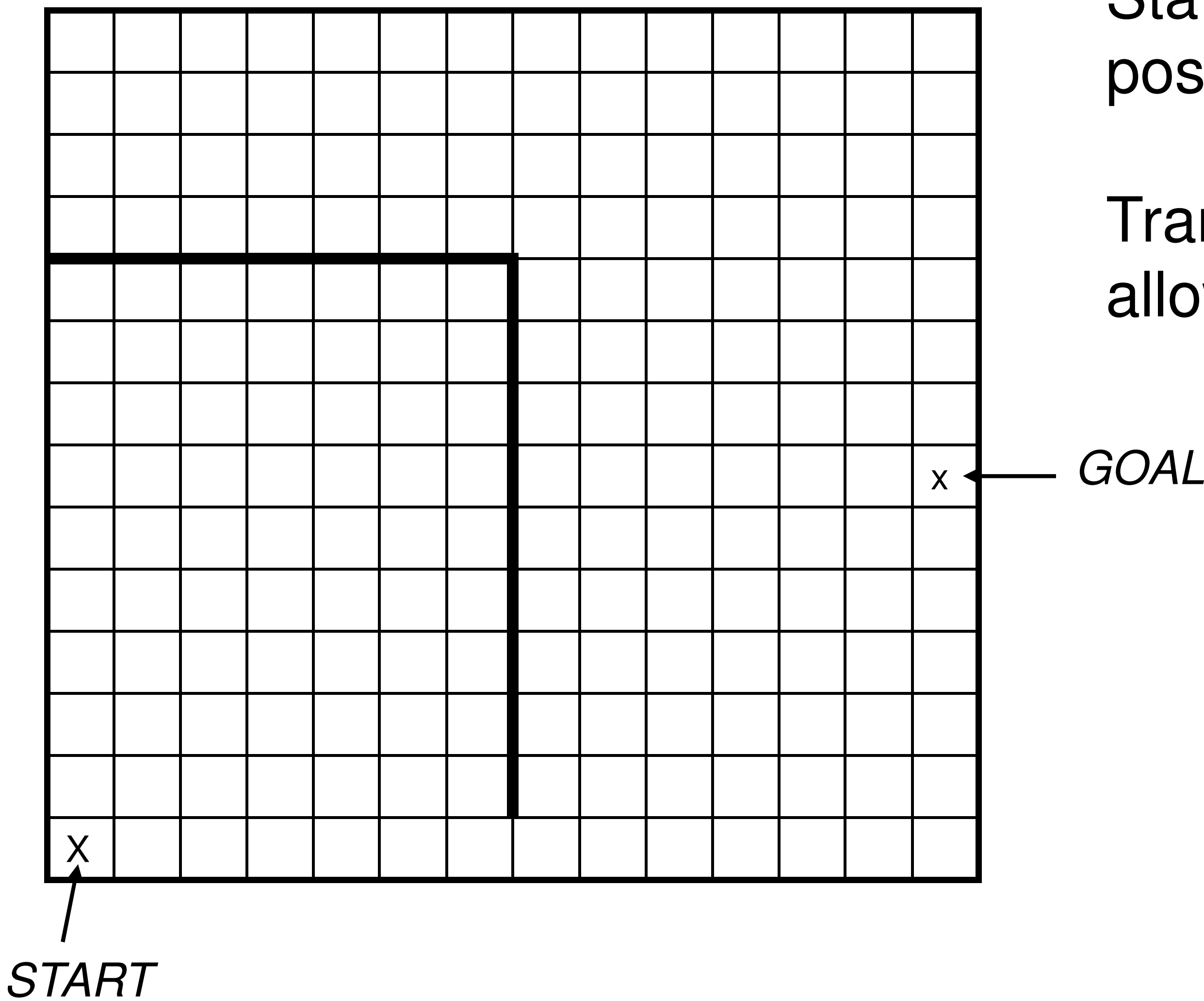


START *GOAL*

- State: Configuration of puzzle
- Transitions: Up to 4 possible moves (*up*, *down*, *left*, *right*)
- Solvable in 22 steps (average)
- But: 1.8 10^5 states (1.3 10^{12} states for the 15-puzzle)

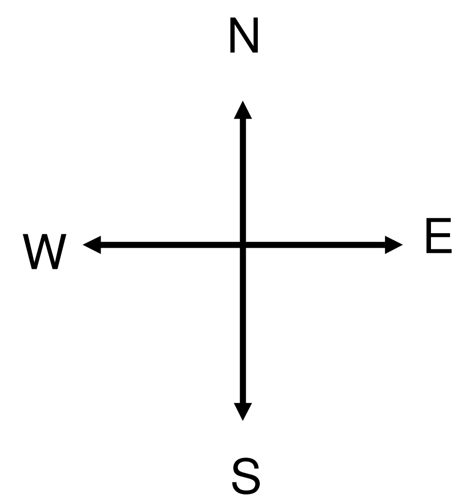
→ Cannot represent set of states explicitly

Example: Robot Navigation



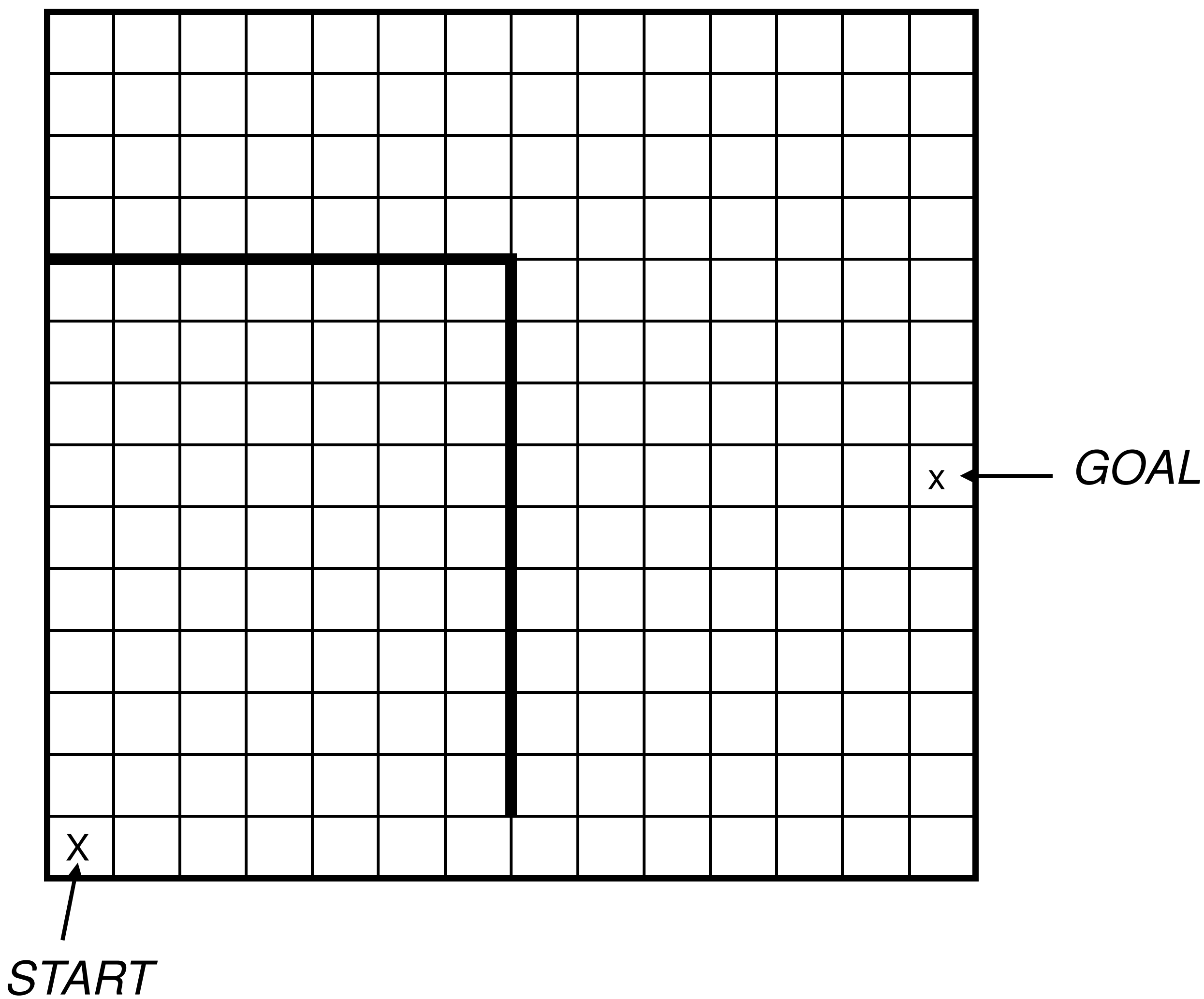
States =
positions in the map

Transitions =
allowed motions

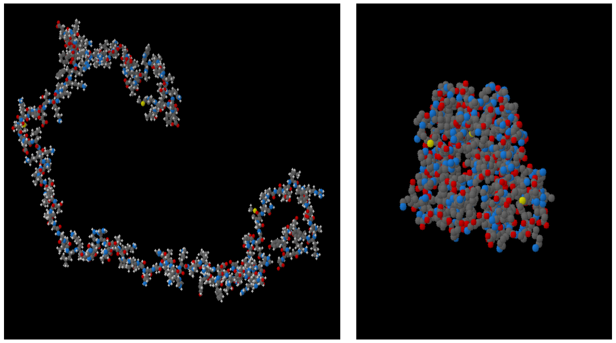


Navigation: Going from point START to point GOAL given a (deterministic) map

Example Solution: Brushfire…

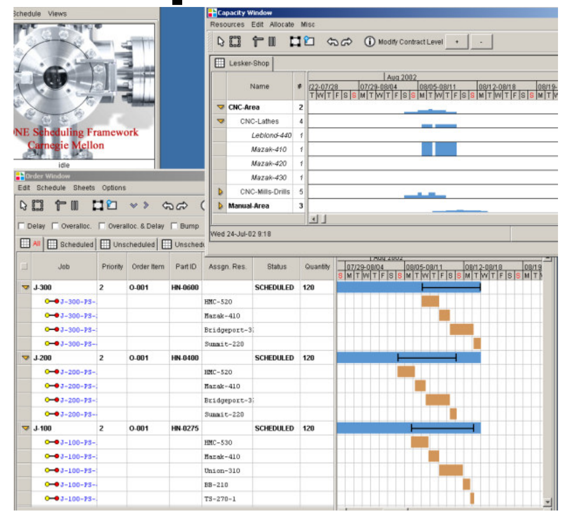


Other Real-Life Examples



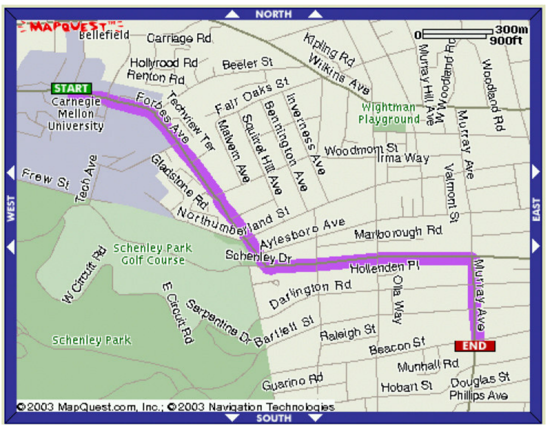
Protein design

http://www.blueprint.org/proteinfolding/trades/trades_problem.html



Scheduling/Manufacturing

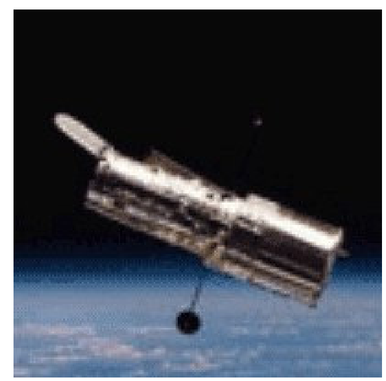
http://www.ozone.ri.cmu.edu/projects/dms/dmsmain.html



Route planning

Robot navigation

http://www.frc.ri.cmu.edu/projects/mars/dstar.html

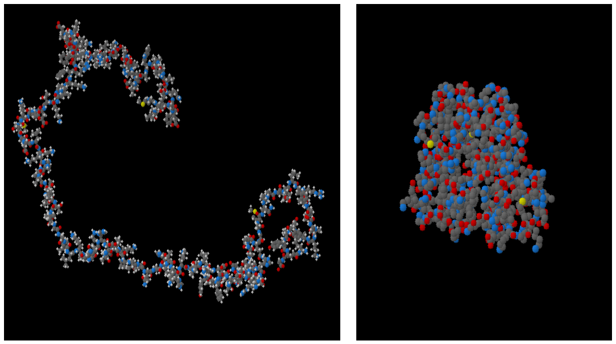


Scheduling/Science

http://www.ozone.ri.cmu.edu/projects/hsts/hstsmain.html

Don’t necessarily know explicitly the structure of a search problem

Other Real-Life Examples



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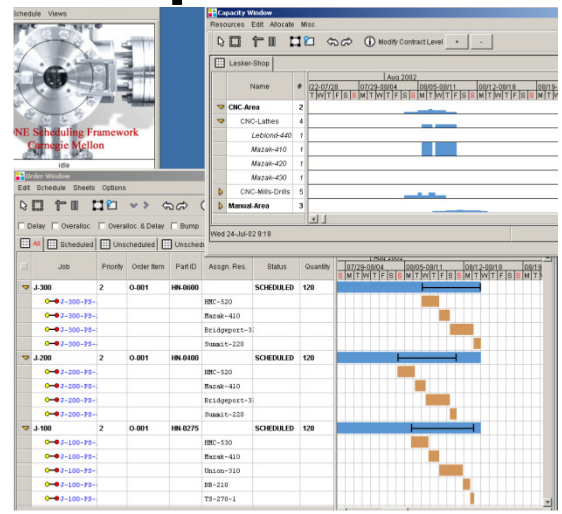

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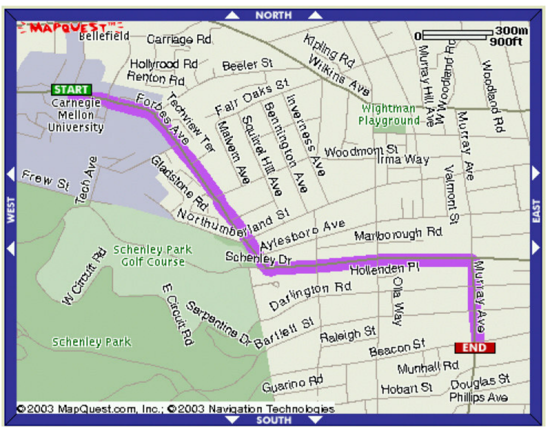

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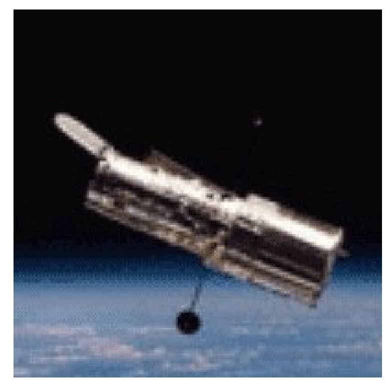
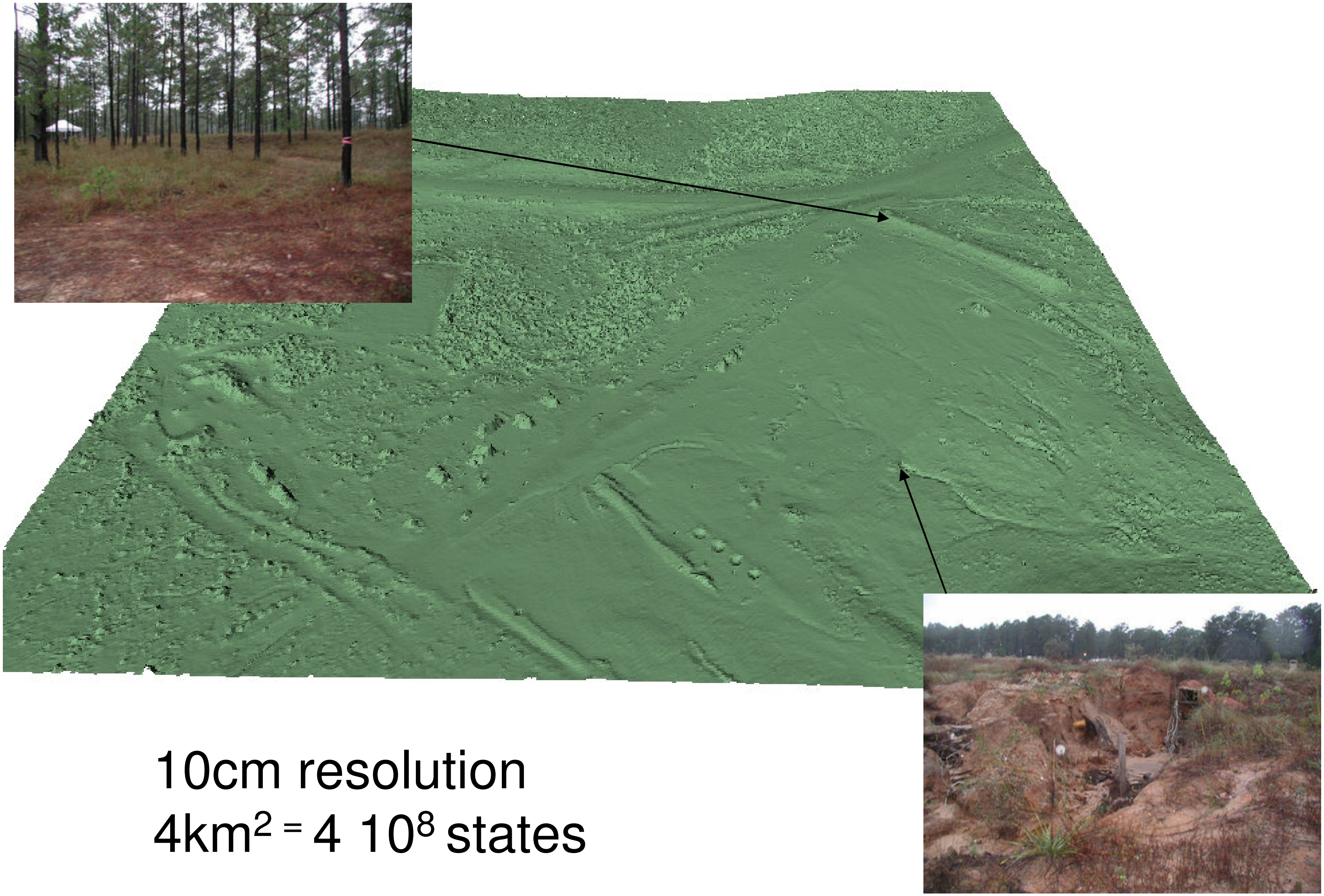


Scheduling/Science

http://www.ozone.ri.cmu.edu/projects/hsts/hstsmain.html

Don’t have a clue when you’re doing well versus poorly!

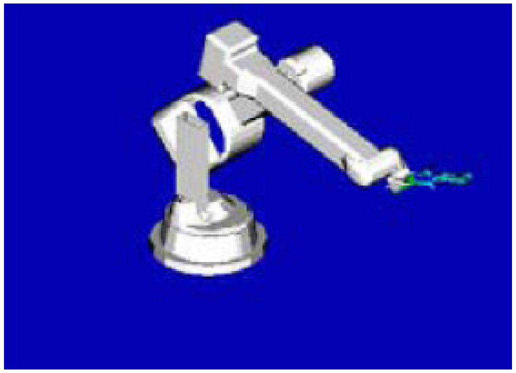


10cm resolution

$4km^2 = 4\ 10^8$ states

What we are *not* addressing (yet)

- Uncertainty/Chance → State and transitions are known and deterministic
- Game against adversary
- Multiple agents/Cooperation
- Continuous state space → For now, the set of states is discrete



Overview

- Definition and formulation
- Optimality, Completeness, and Complexity
- *Uninformed Search*
  - Breadth First Search
  - Search Trees
  - Depth First Search
  - Iterative Deepening
- *Informed Search*
  - Best First Greedy Search
  - Heuristic Search, A*

A Search Problem: Square World

Formulation

- Q: Finite set of states
- $S \subseteq Q$: Non-empty set of start states
- $G \subseteq Q$: Non-empty set of goal states
- **succs**: function $Q \rightarrow \mathcal{P}(Q)$
  succs(s) = Set of states that can be reached from s in one step
- **cost**: function Qx$Q \rightarrow$ Positive Numbers
  cost(s,s') = Cost of taking a one-step transition from state s to state s'

- Problem: Find a sequence $\{s_1,\ldots,s_K\}$ such that:

1. $s_1 \in S$
2. $s_K \in G$
3. $s_{i+1} \in$ **succs**(s_i)
4. $\sum$ **cost**(s_i, s_{i+1}) is the smallest among all possible sequences (desirable but optional)

What about actions?

- Q: Finite set of states
- $S \subseteq Q$: Non-empty set of start states
- $G \subseteq Q$: Non-empty set of goal states
- **succs**: function $Q \rightarrow \mathcal{P}(Q)$
  succs(s) = Set of states that can be reached from s in one step
- **cost**: function Qx$Q \rightarrow$ Positive Numbers
  cost(s,s') = Cost of taking a one-step transition from state s to state s'

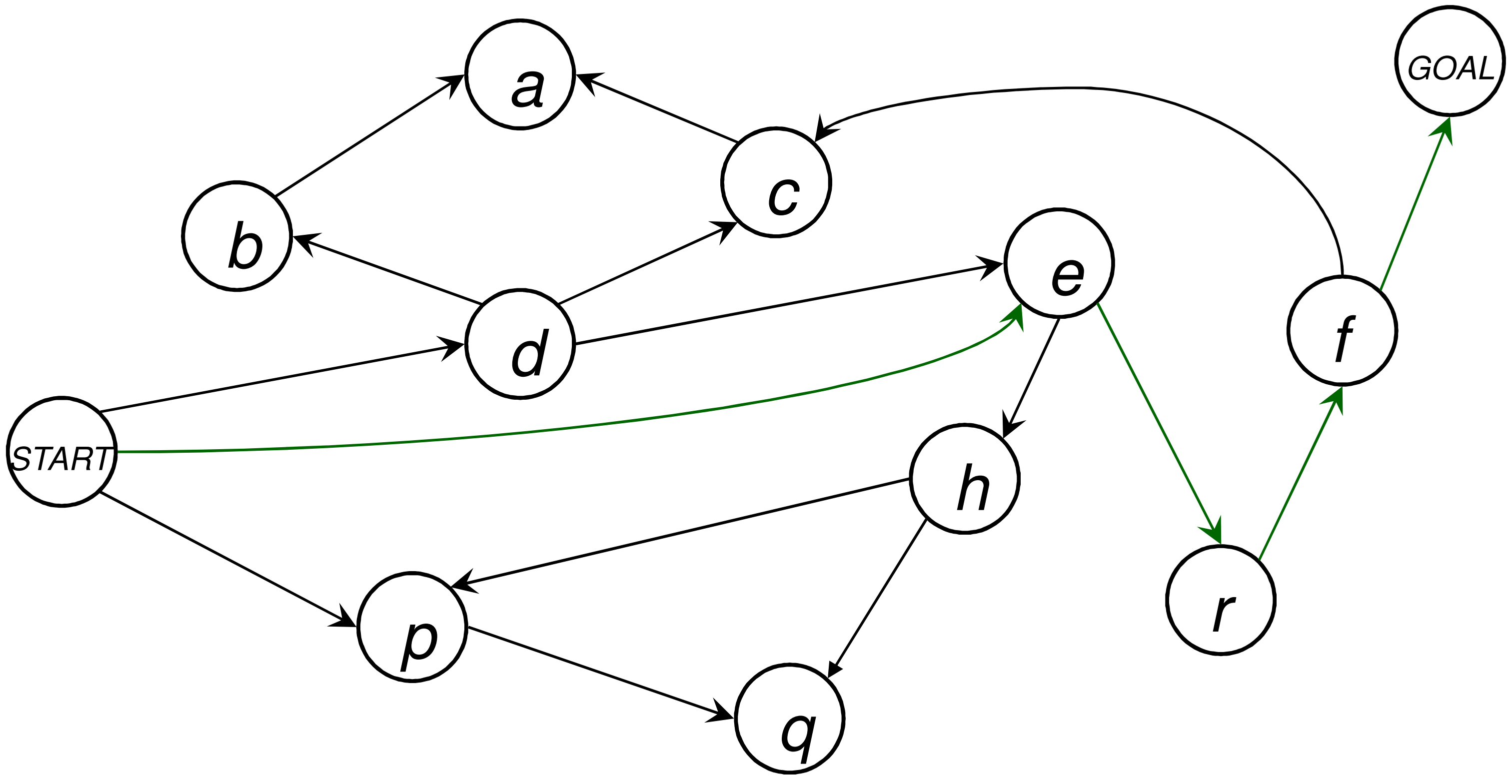
- Problem: Find a sequence $\{s_1,\ldots,s_K\}$ such that:

Actions define transitions from states to states.
Example: Square World

Example

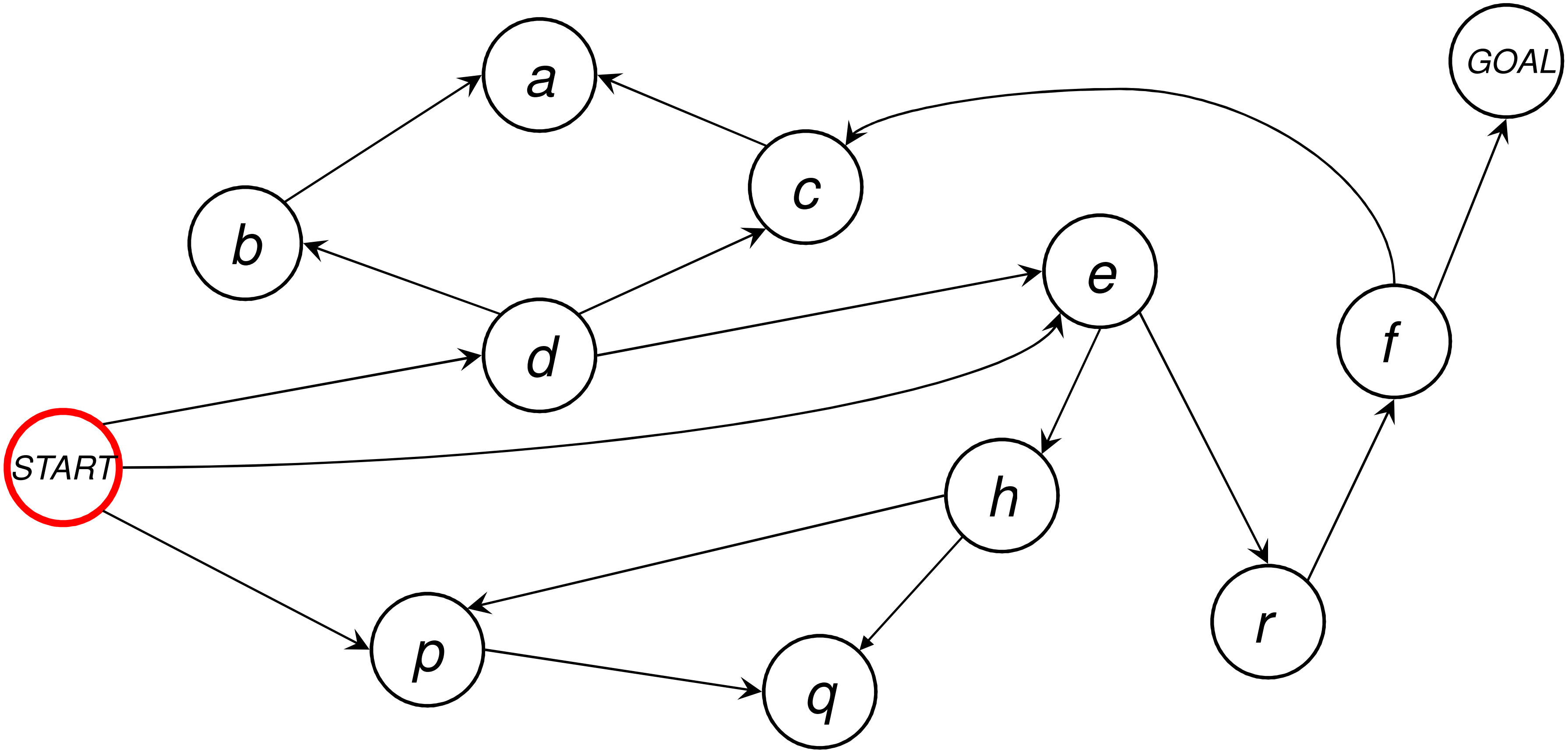
- $Q = \{AA, AB, AC, AD, AI, BB, BC, BD, BI, \ldots\}$
- $S = \{AB\}$ $G = \{DD\}$
- **succs**(AA) = $\{AI, BA\}$
- **cost**(s,s') = 1 for each action (transition)

Desirable Properties



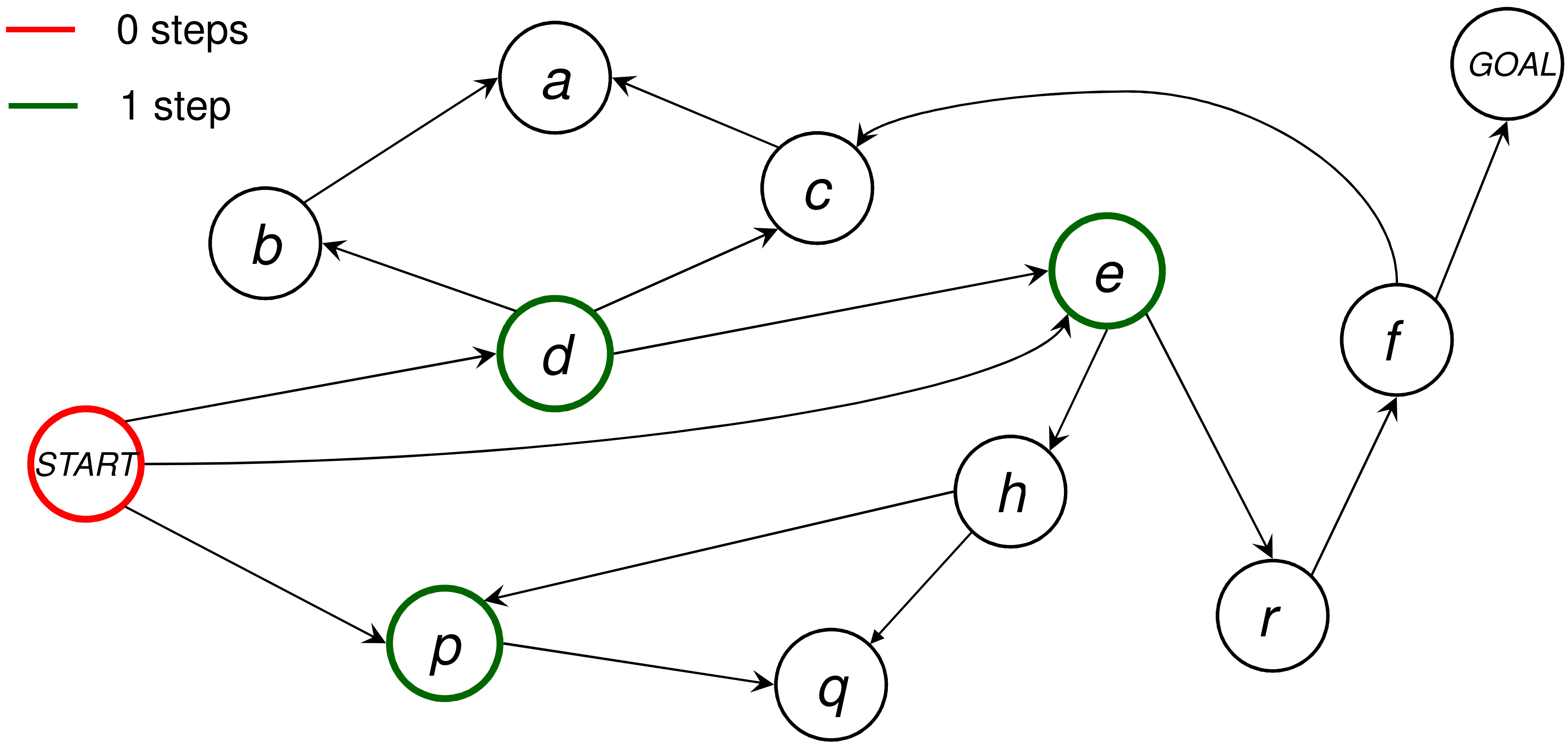
- ***Completeness*: An algorithm is complete if it is guaranteed to find a path if one exists**
- ***Optimality*: The total cost of the path is the lowest among all possible paths from start to goal**
- ***Time Complexity***
- ***Space Complexity***

Breadth-First Search



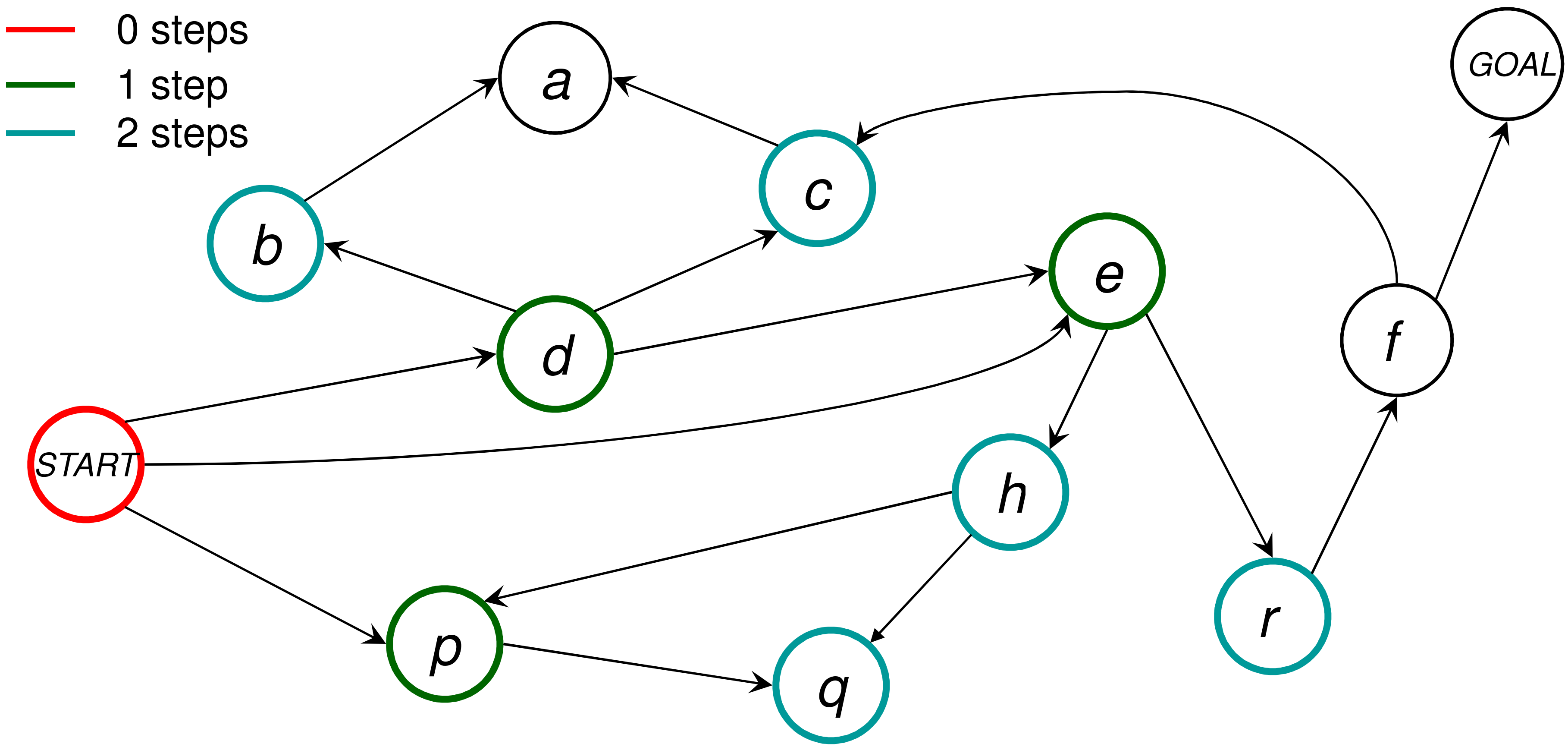
- Label all states that are 0 steps from S → Call that set V_o

Breadth-First Search



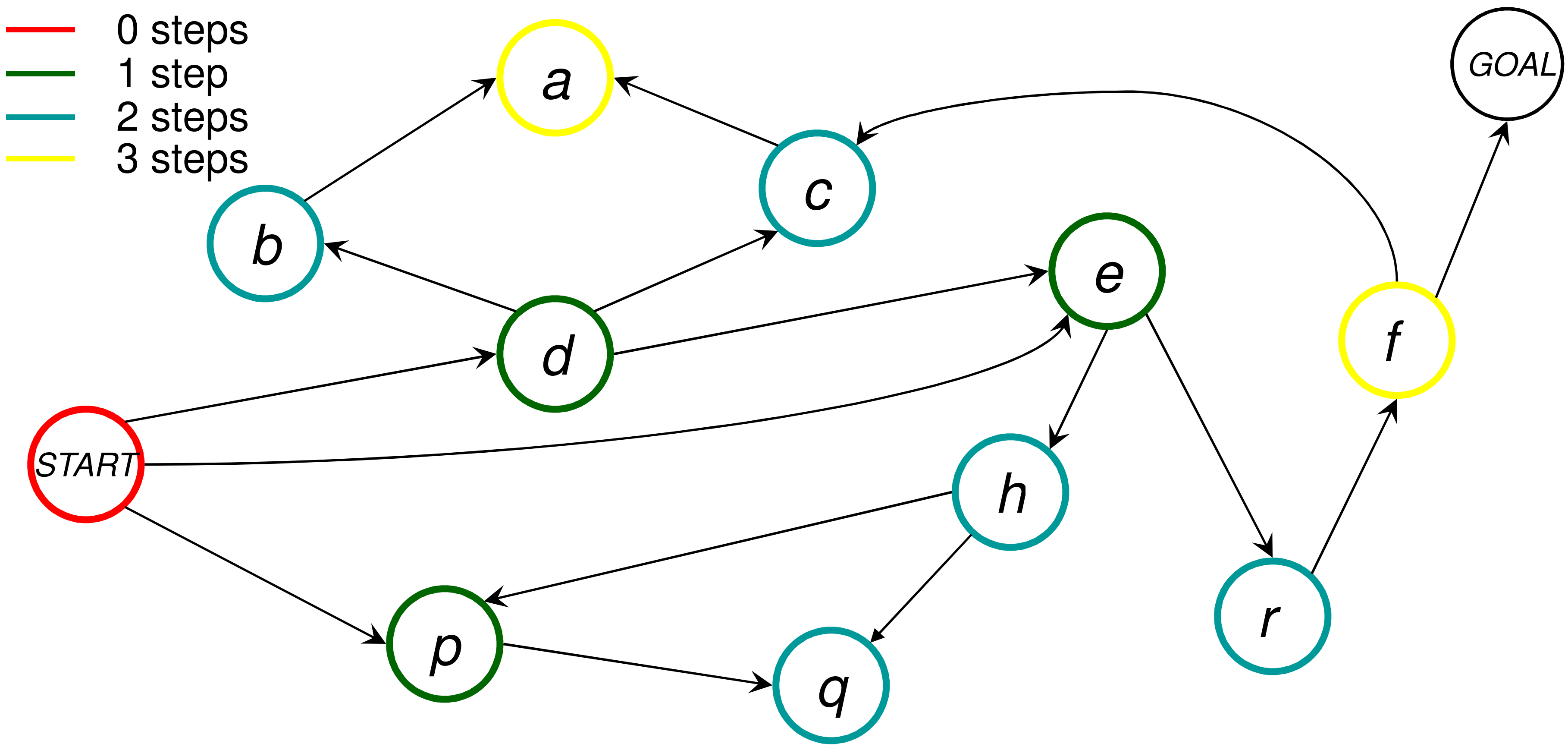
- Label the successors of the states in V_o that are not yet labelled → Set V_1 of states that are 1 step away from the start

Breadth-First Search



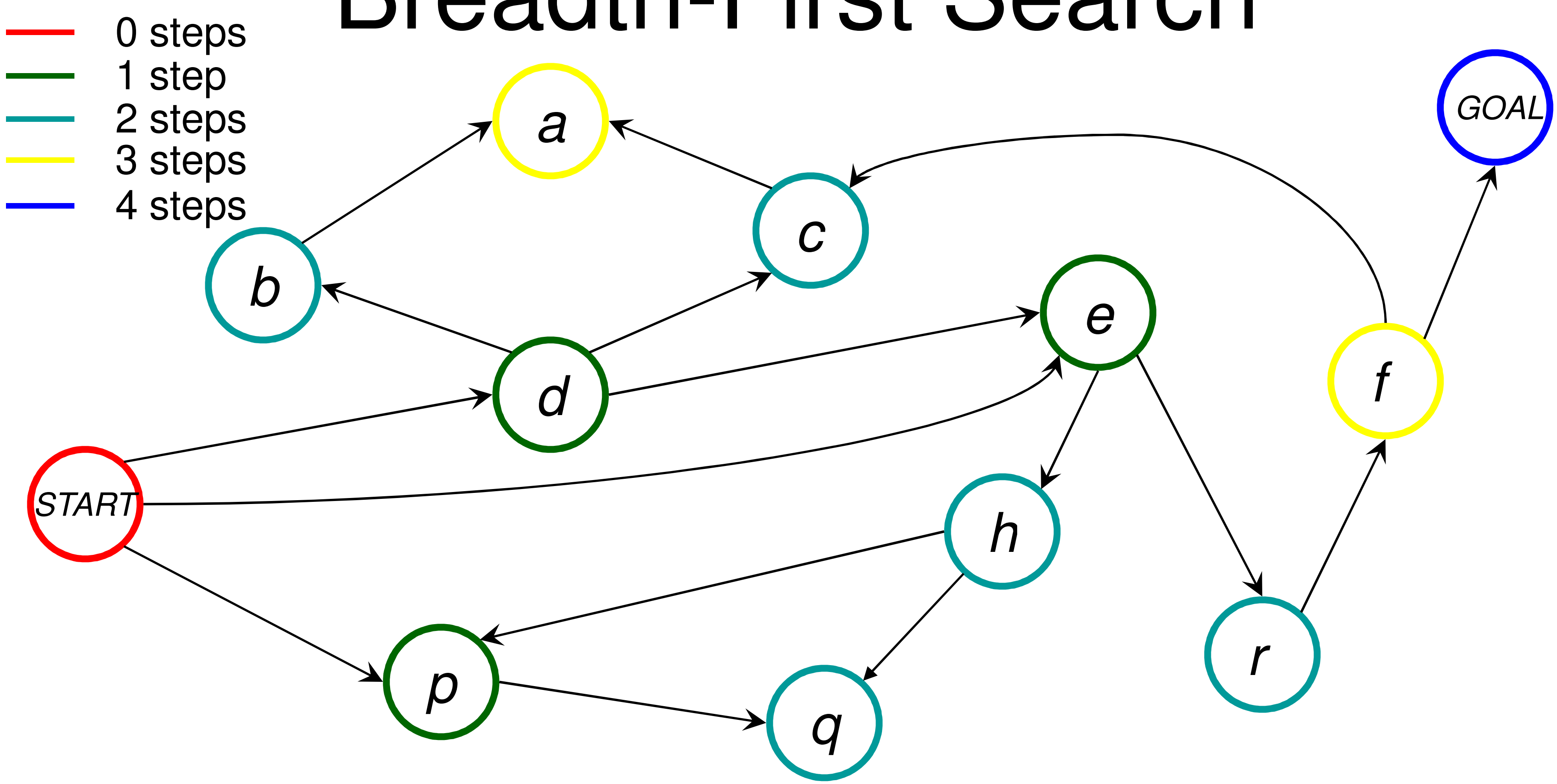
- Label the successors of the states in V_1 that are not yet labelled →Set V_2 of states that are 1 step away from the start

Breadth-First Search



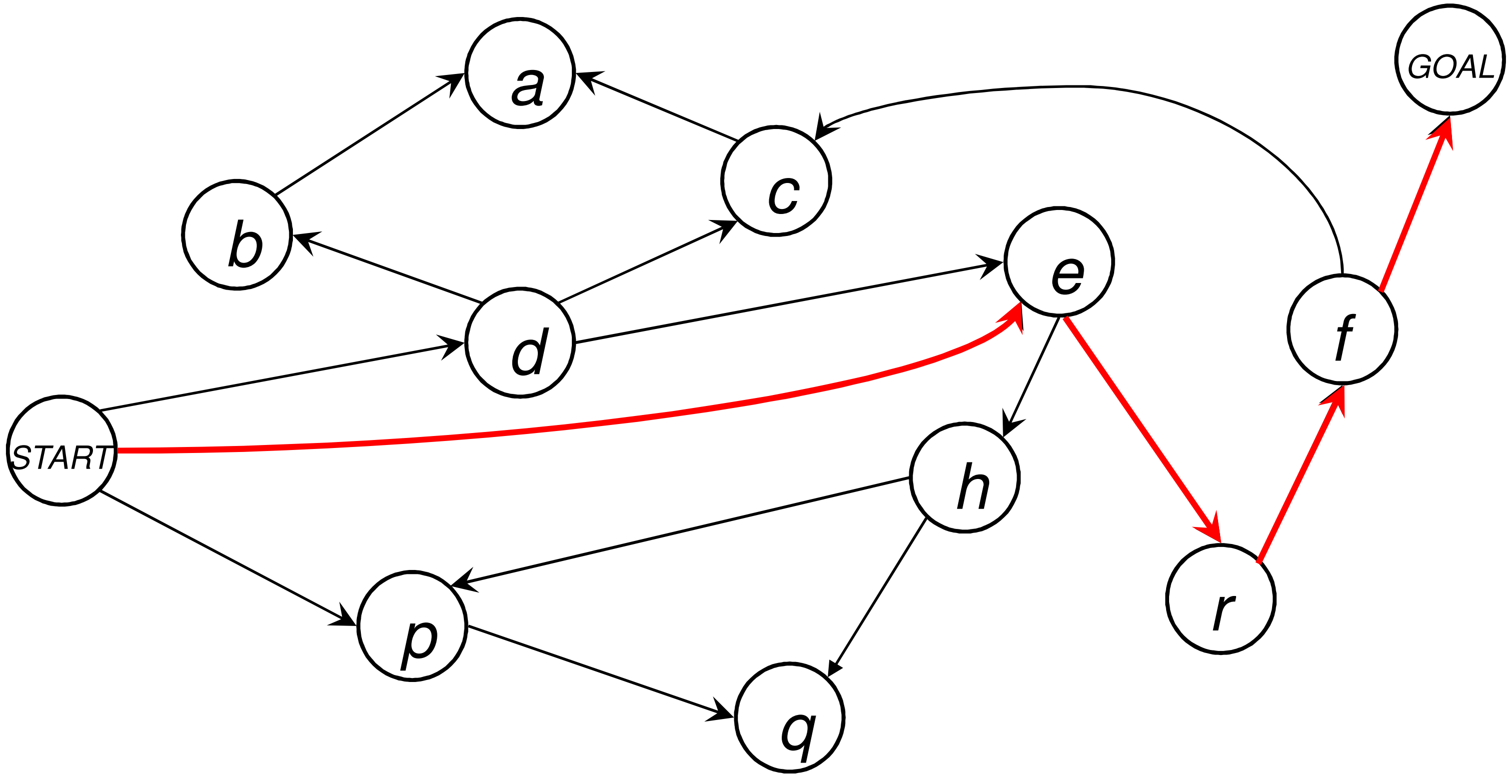
- Label the successors of the states in V_2 that are not yet labelled →Set V_3 of states that are 1 step away from the start

Breadth-First Search



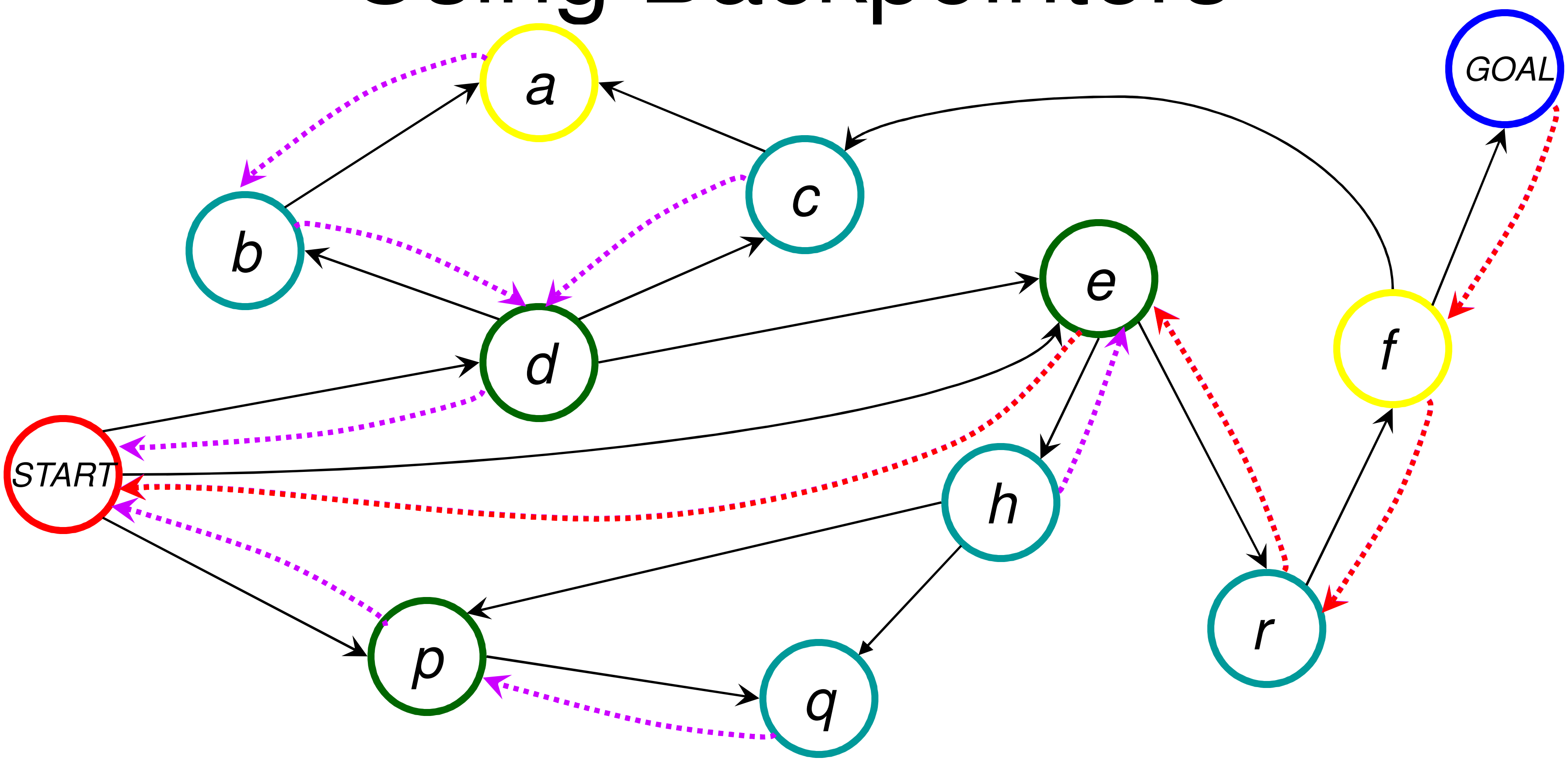
- Stop when goal is reached in the current expansion set → goal can be reached in 4 steps

Recovering the Path



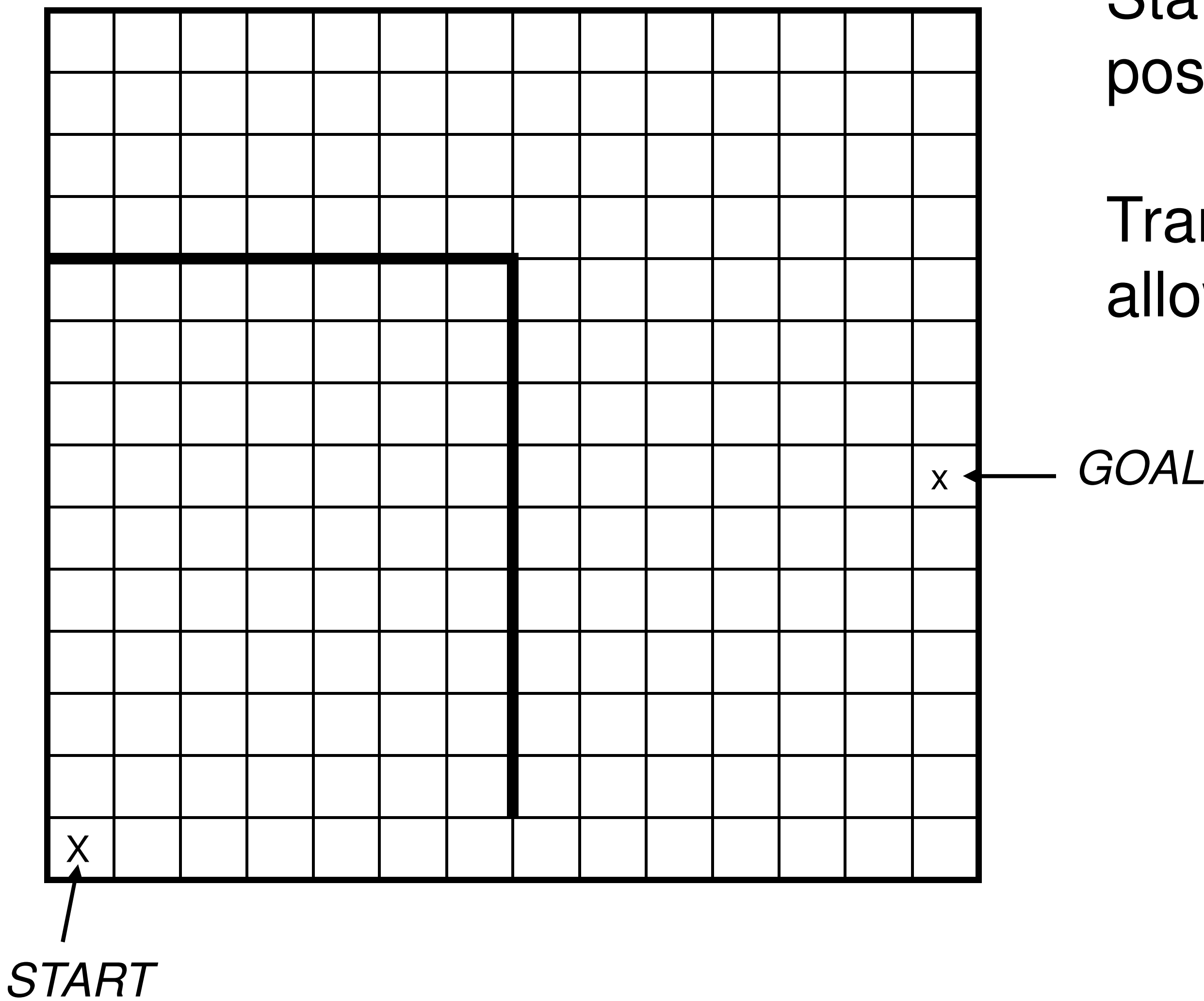
- Record the predecessor state when labeling a new state
- When I labeled *GOAL*, I was expanding the neighbors of *f* so therefore *f* is the predecessor of *GOAL*
- When I labeled *f*, I was expanding the neighbors of *r* so therefore *r* is the predecessor of *f*
- Final solution: {*START, e, r, f, GOAL*}

Using Backpointers



- A backpointer **previous**(*s*) points to the node that stored the state that was expanded to label *s*
- The path is recovered by following the backpointers starting at the goal state

Example: Robot Navigation



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positions in the map

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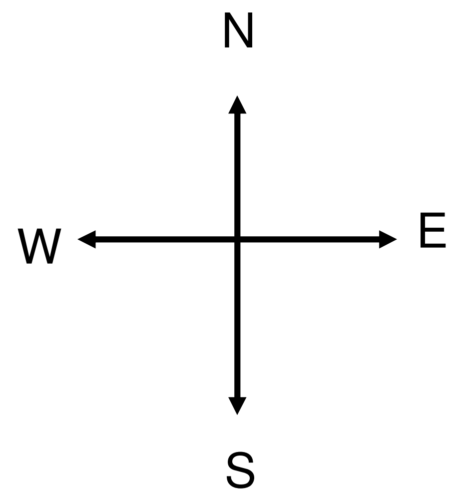

Navigation: Going from point START to point GOAL given a (deterministic) map

Breadth First Search

$V_0 \leftarrow S$ (the set of start states)

previous(*START*) := *NULL*

$k \leftarrow 0$

while (V_k is not a subset of the goal set *and* V_k is not empty) **do**

- $V_{k+1} \leftarrow$ empty set
- For each state *s* in V_k
  - For each state *s'* in **succs**(*s*)
    - If *s'* has not already been labeled
      - Set **previous**(*s'*) $\leftarrow$ *s*
      - Add *s'* into V_{k+1}
- $k \leftarrow k+1$

if V_k is empty signal FAILURE

else build the solution path thus:

- Define S_k = *GOAL*, and forall $i <= k$, define S_{i-1} = **previous**(S_i)
- Return *path* = {S … S }

Properties

- BFS can handle multiple start and goal states **what does multiple start mean?**
- Can work either by searching forward from the start or backward for the goal (forward/backward chaining)
- (Which way is better?)
- Guaranteed to find the lowest-cost path in terms of number of transitions??

See maze example

Complexity

- N = Total number of states
- B = Average number of successors (branching factor)
- L = Length from start to goal with smallest number of steps

Algorithm		Complete	Optimal	Time	Space
BFS	Breadth First Search				

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- N = Total number of states
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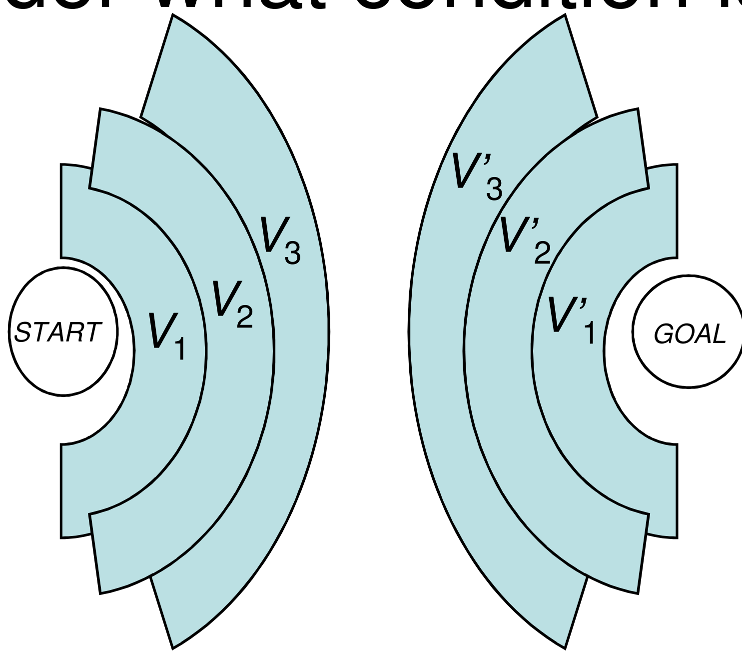
Algorithm		Complete	Optimal	Time	Space
BFS	Breadth First Search	Y	Y, If all trans. have same cost	$O(\min(N,B^L))$	$O(\min(N,B^L))$

Bidirectional Search

- BFS search simultaneously forward from *START* and backward from *GOAL*
- When do the two search meet?
- What stopping criterion should be used?
- Under what condition is it optimal?



Complexity

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Algorithm		Complete	Optimal	Time	Space
BFS	Breadth First Search				
BIBFS	Bi-directional Breadth First Search				

Major savings when bidirectional search is possible because

$$2B^{L/2} << B^L$$

B = 10, L = 6 → 22,200 states generated vs. $\sim 10^7$

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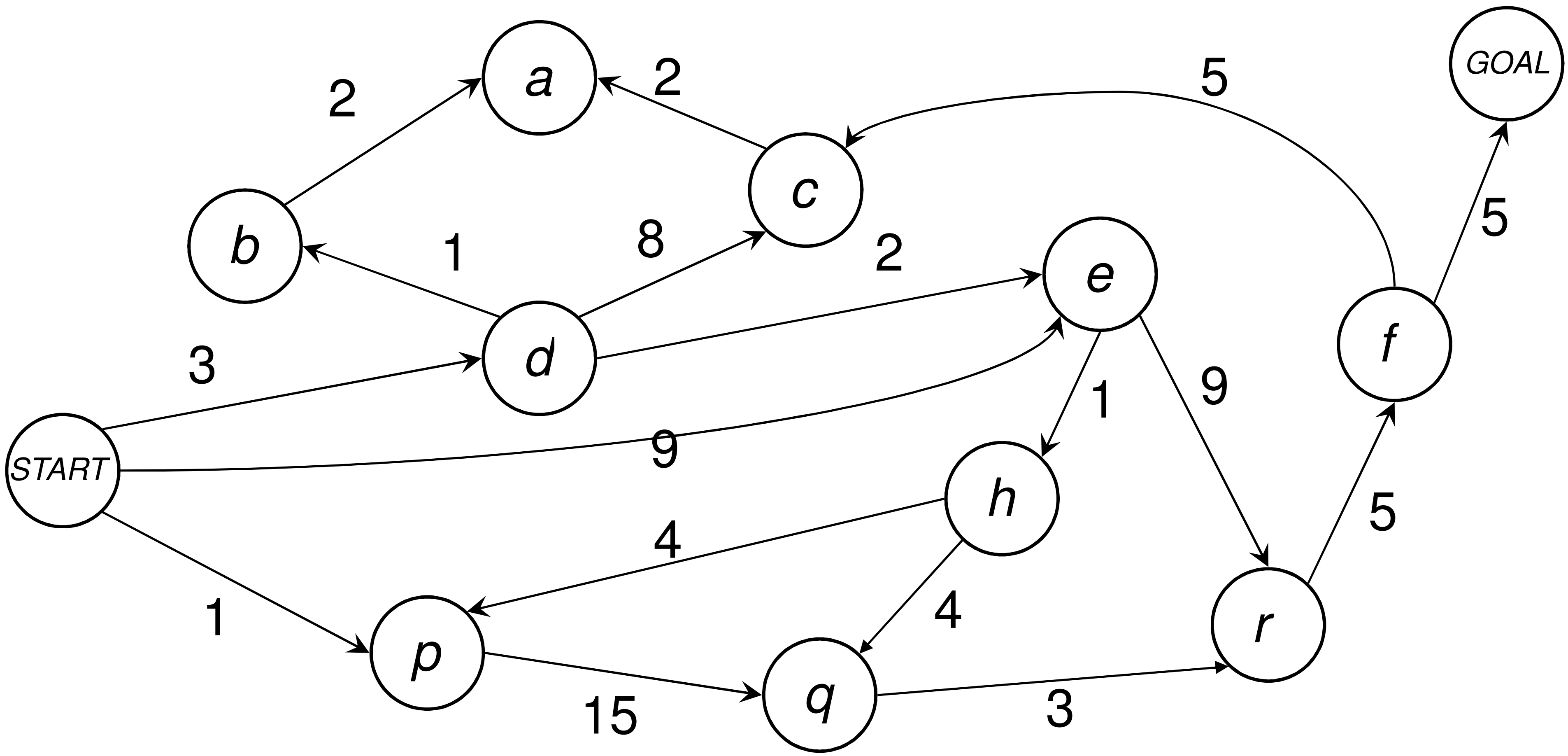
B = 10, L = 6 → 22,200 states generated vs. $\sim 10^7$

Complexity

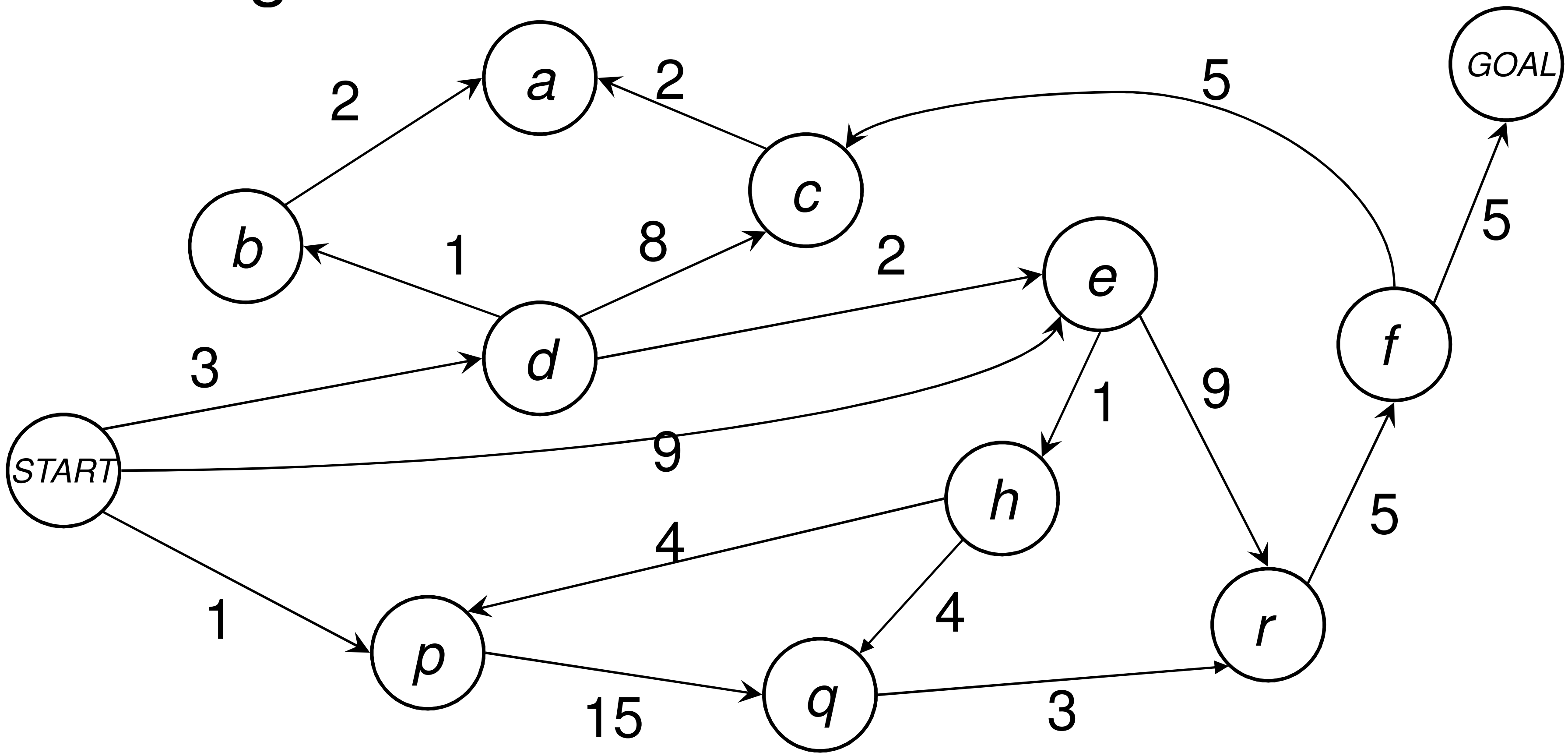
- *A note about island-driven search in general:*
  - What happens to complexity if you have L islands enroute to the goal?

Algorithm		Complete	Optimal	Time	Space
BFS	Breadth First Search	Y	Y, if all trans. have same cost	$O(\min(N,B^L))$	$O(\min(N,B^L))$
BIBFS	Bi-directional Breadth First Search	Y	Y, If all trans. have same cost	$O(\min(N,2B^{L/2}))$	$O(\min(N,2B^{L/2}))$

Counting Transition Costs Instead of Transitions



Counting Transition Costs Instead of Transitions



- BFS finds the shortest path in number of steps but does not take into account transition costs
- Simple modification finds the least cost path
- New field: At iteration k, $\mathbf{g}(s)$ = least cost path to s in k or fewer steps

Uniform Cost Search

- Strategy to select state to expand next
- Use the state with the smallest value of **g**() so far
- Use priority queue for efficient access to minimum **g** at every iteration

Priority Queue

- Priority queue = data structure in which data of the form (*item*, *value*) can be inserted and the item of minimum value can be retrieved efficiently
- Operations:
  - **Init** (*PQ*): Initialize empty queue
  - **Insert** (*PQ*, *item*, *value*): Insert a pair in the queue
  - **Pop** (*PQ*): Returns the pair with the minimum *value*
- In our case:
  - *item* = state   *value* = current cost **g**()

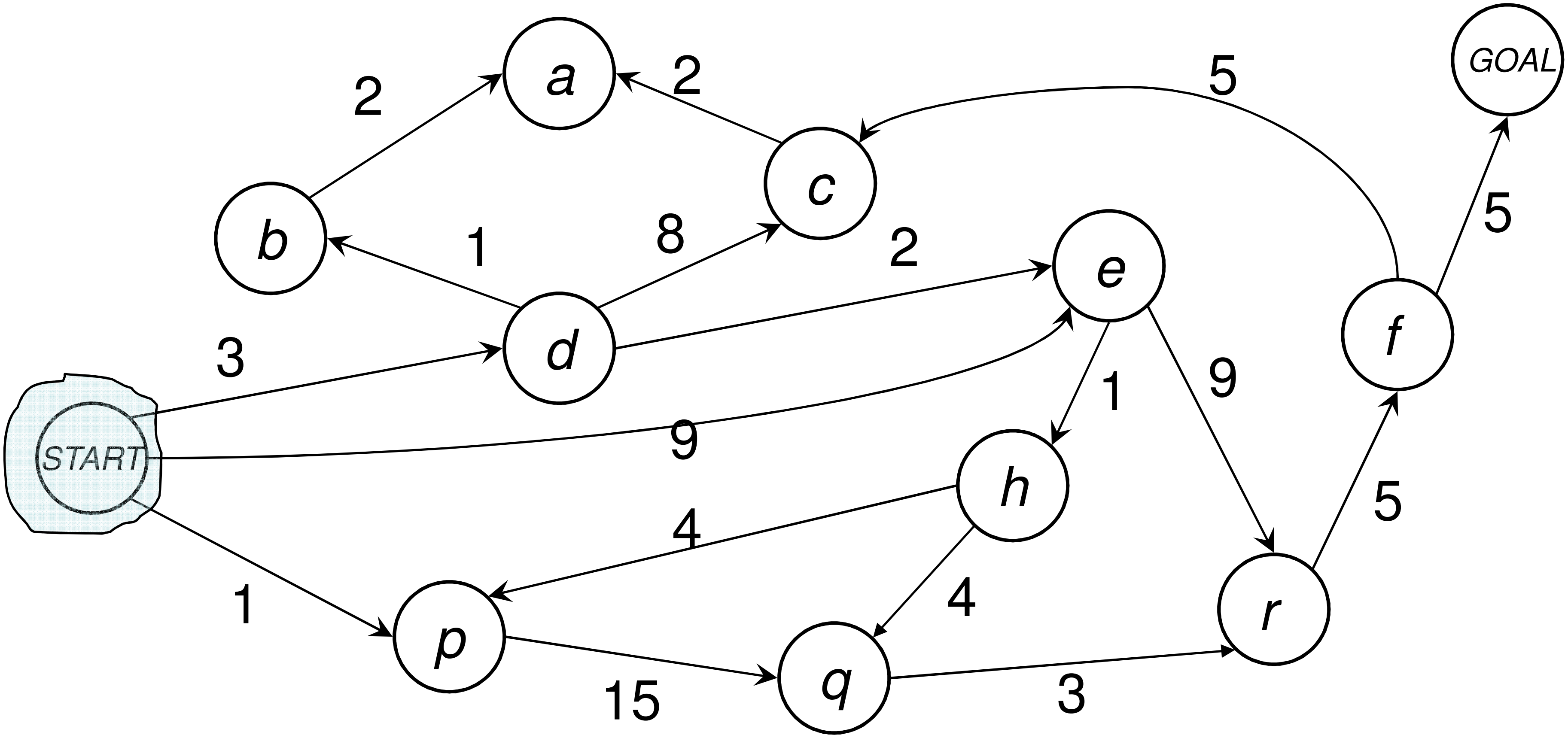
Complexity: O(log(number of pairs in PQ)) for insertion and pop operations → very efficient

http://www.leekillough.com/heaps/   Knuth&Sedwick ….

Uniform Cost Search

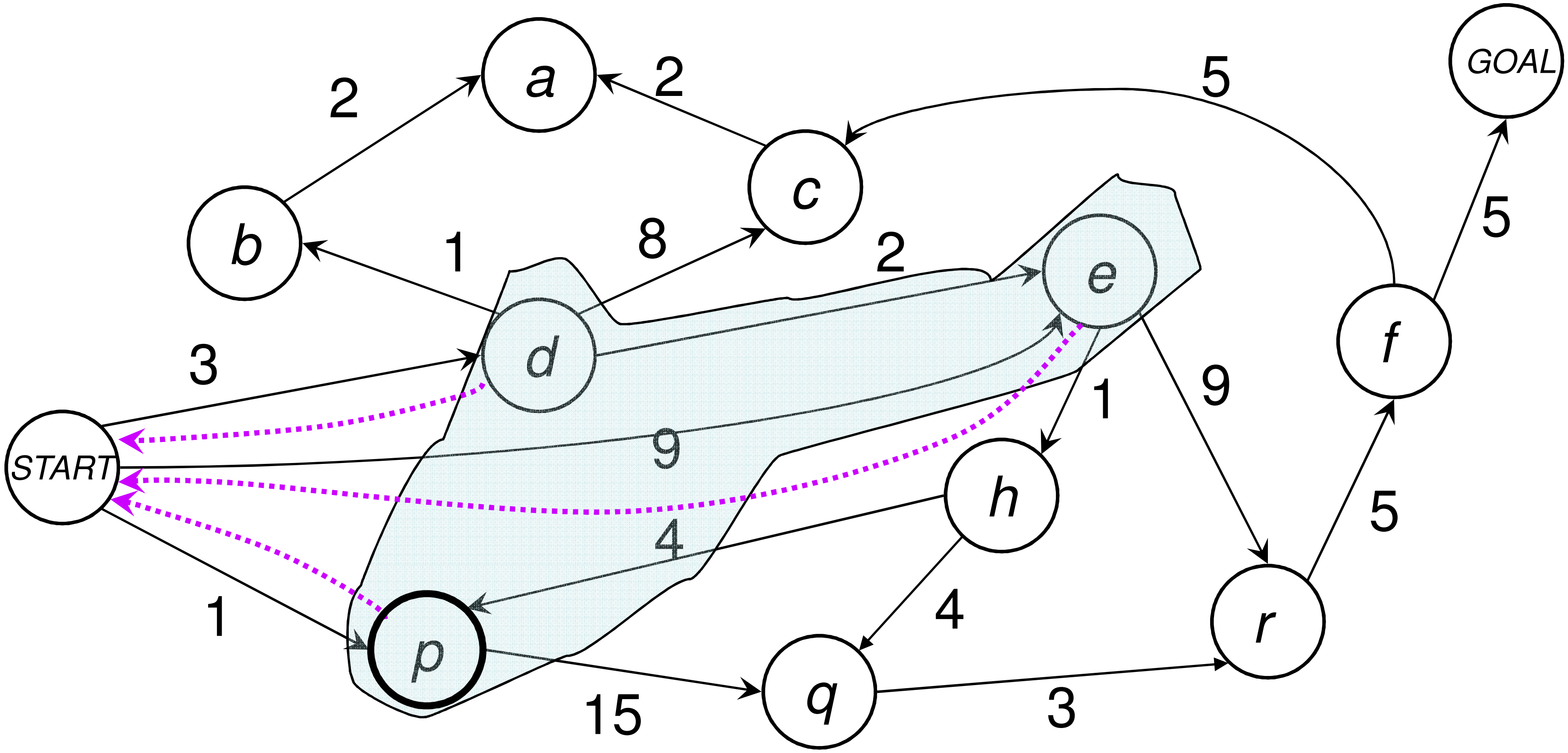
- *PQ* = Current set of evaluated states
- Value (priority) of state = **g**(*s*) = current cost of path to *s*
- Basic iteration:
  1. Pop the state *s* with the lowest path cost from *PQ*
  2. Evaluate the path cost to all the successors of *s*
  3. Add the successors of *s* to *PQ*

We add the successors of *s* that have not yet been visited and we update the cost of those currently in the queue



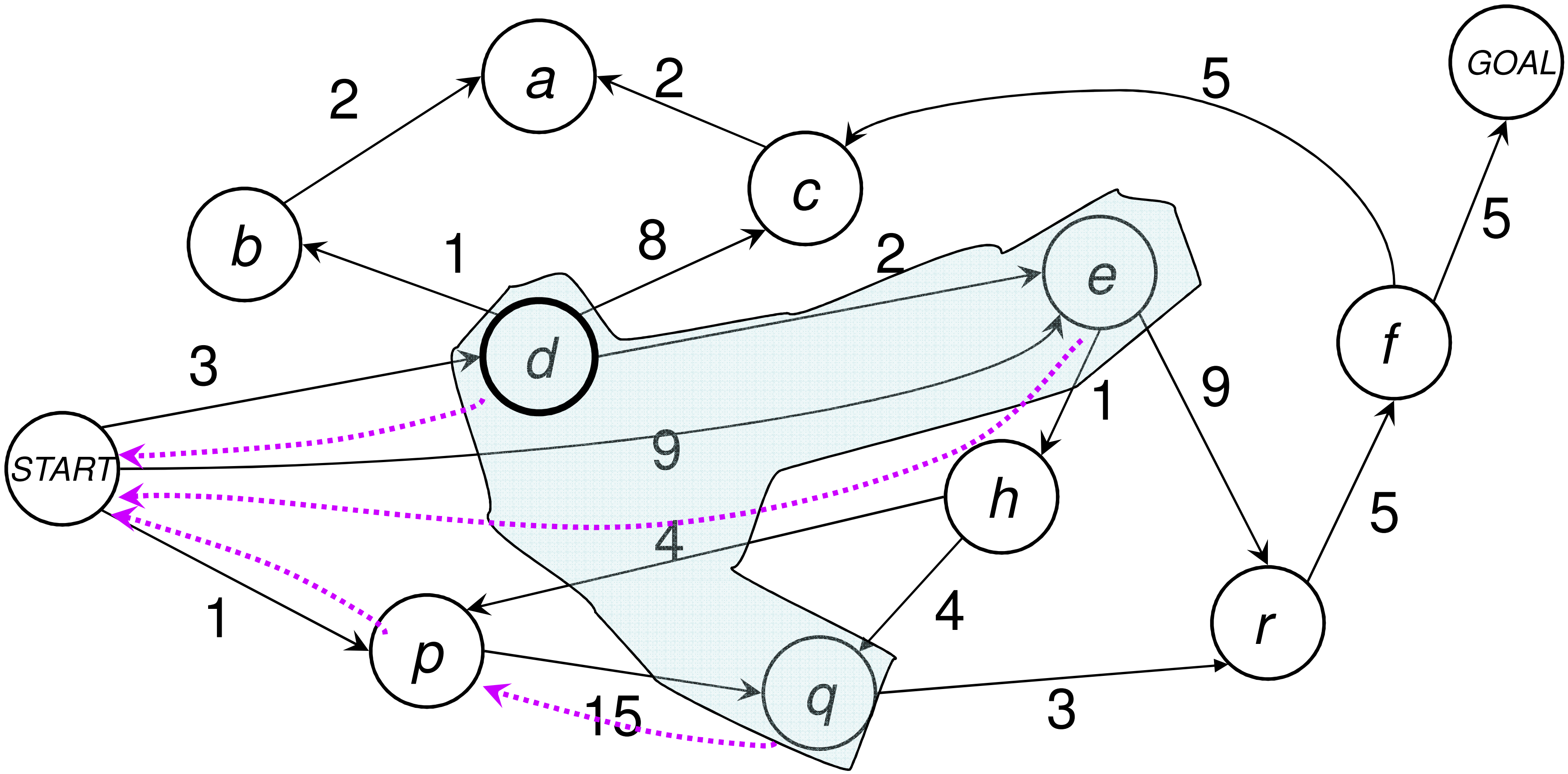
$PQ = \{(START,0)\}$

1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* to *PQ*



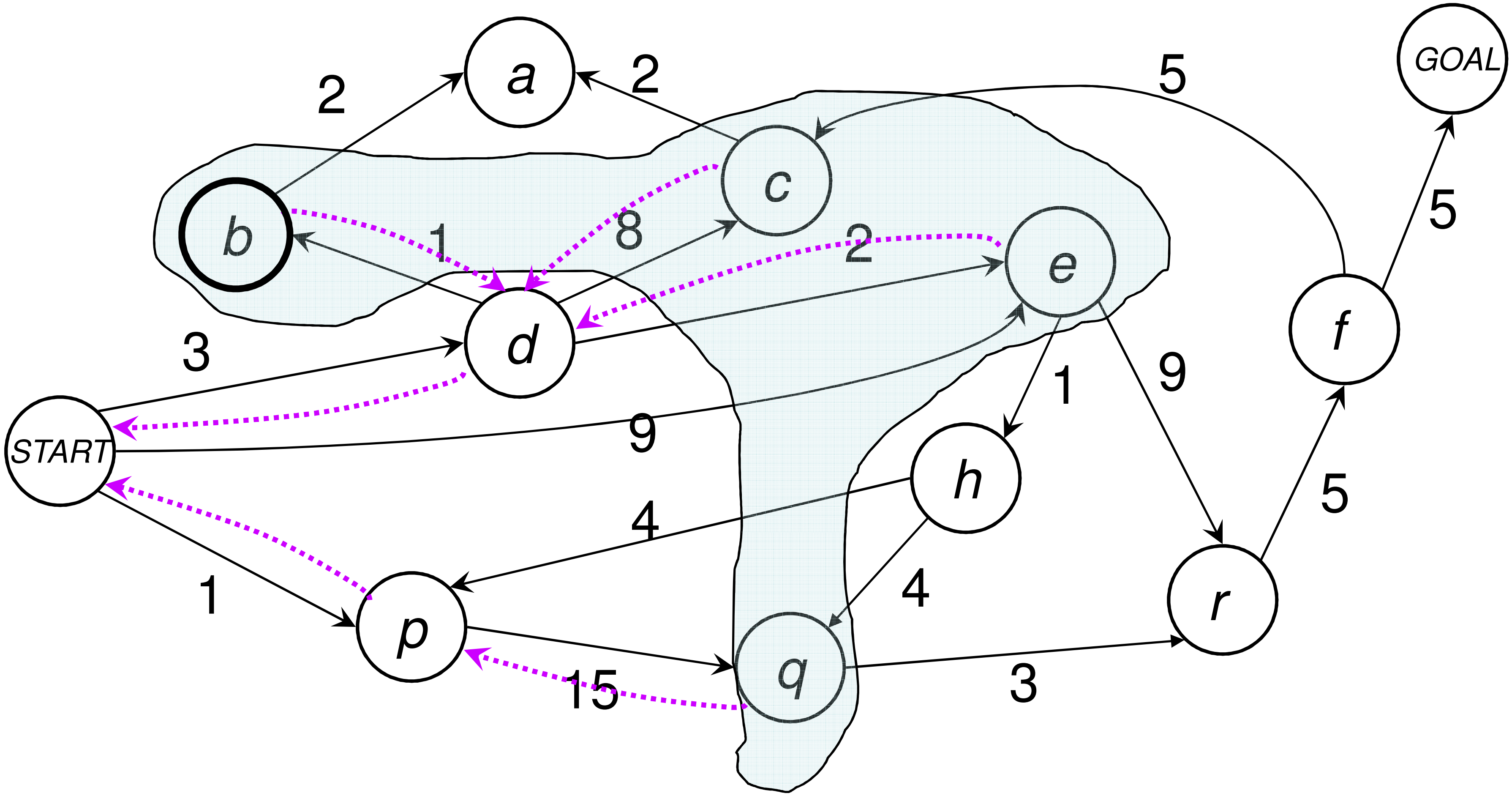
$PQ = \{(p,1)\ (d,3)\ (e,9)\}$

1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* to *PQ*



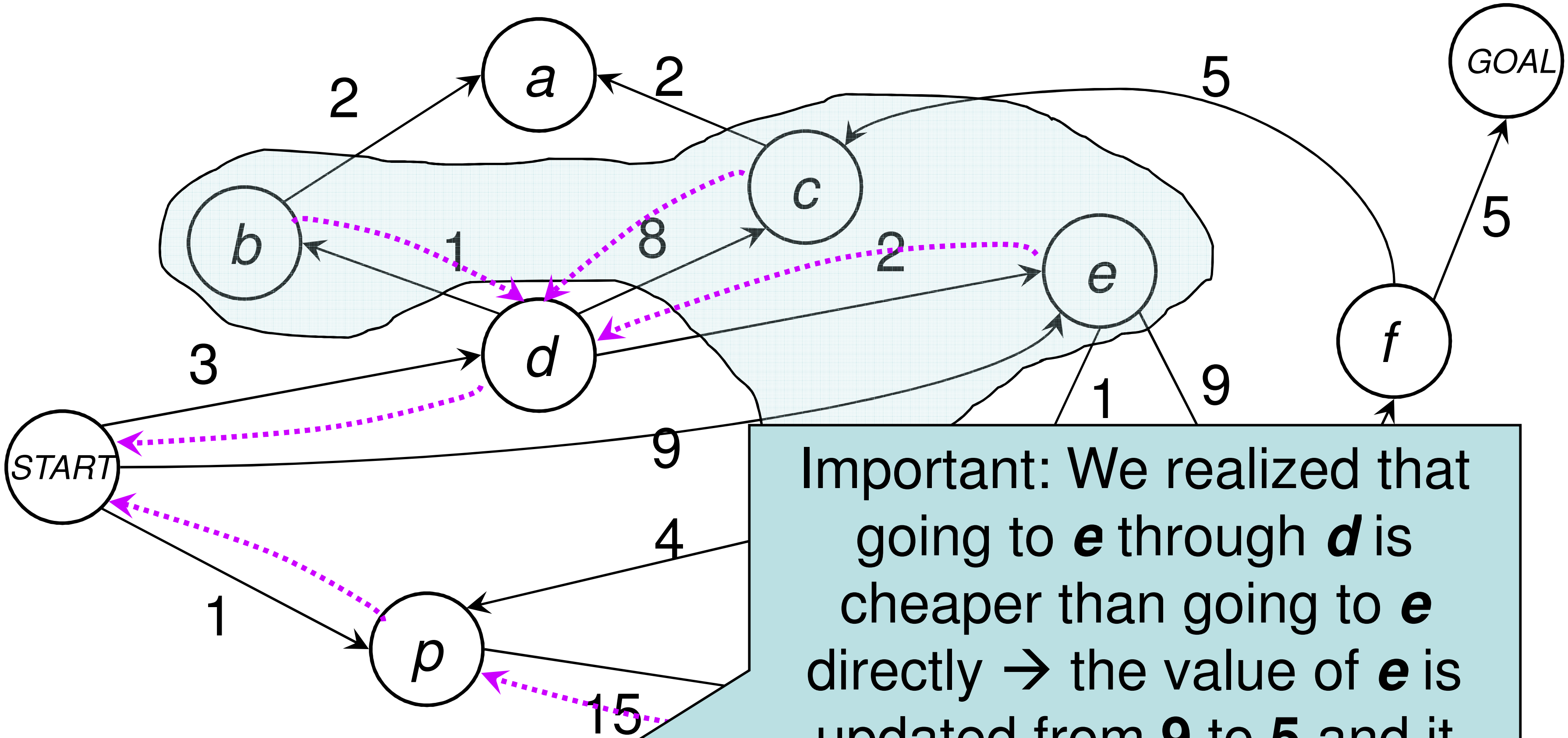
PQ = {(d,3) (e,9) (q,16)}

1. Pop the state s with the lowest path cost from PQ
2. Evaluate the path cost to all the successors of s
3. Add the successors of s to PQ



PQ = {(b,4) (e,5) (c,11) (q,16)}

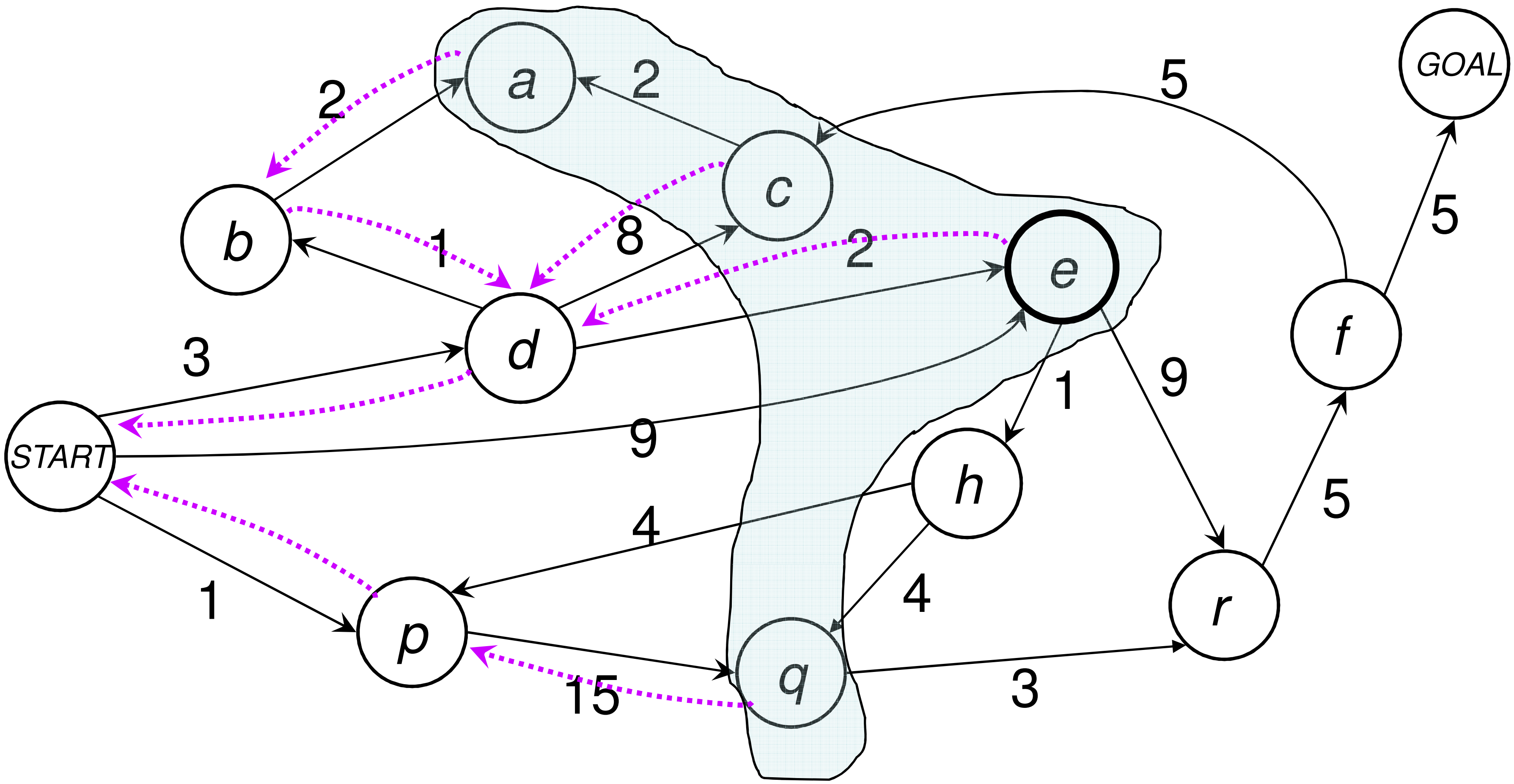
1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* tc *PQ*



Important: We realized that going to ***e*** through ***d*** is cheaper than going to ***e*** directly → the value of ***e*** is updated from **9** to **5** and it moves up in PQ

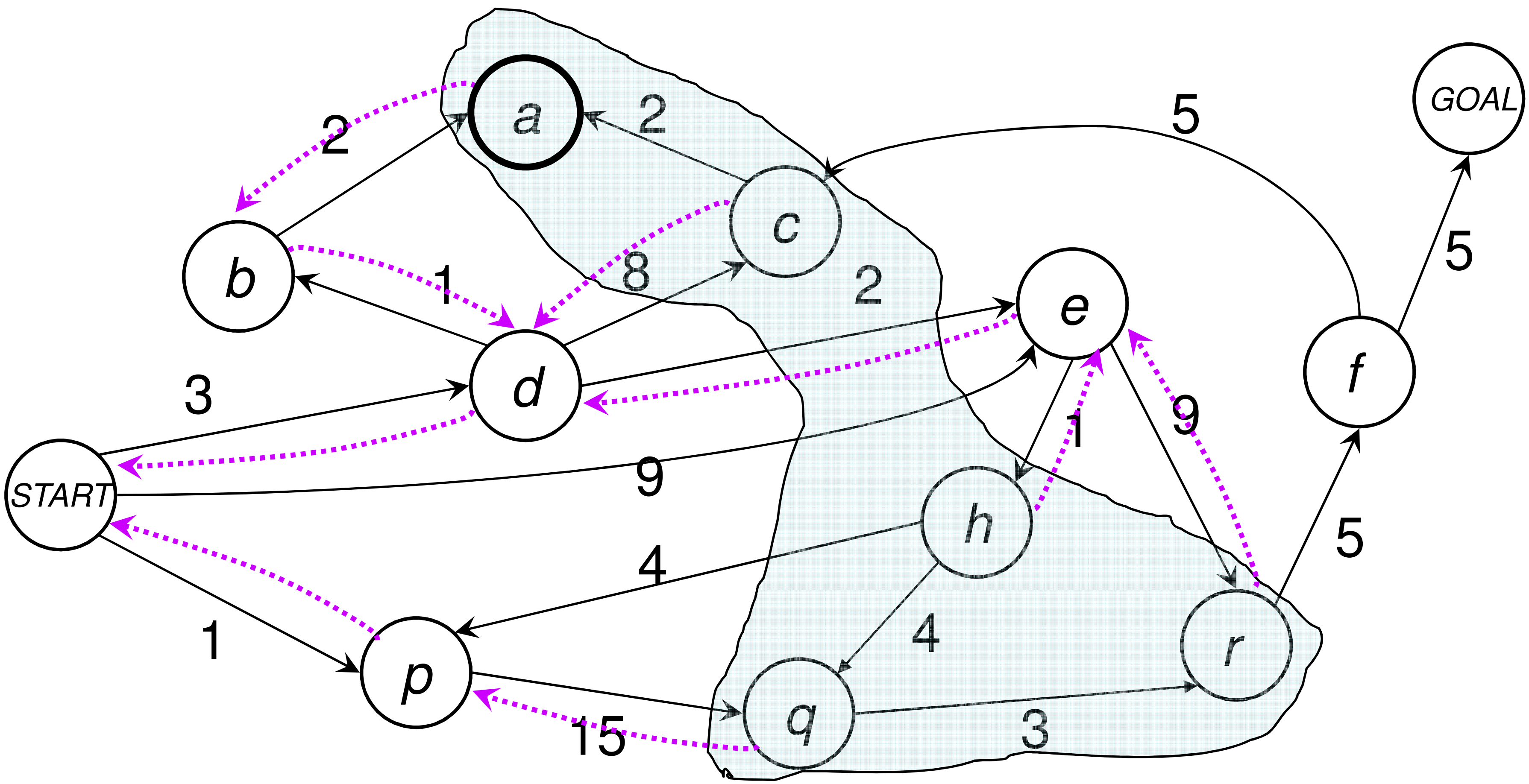
***PQ* = {(*b*,4) (*e*,5) (*c*,11) (*q*,16)}**

1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* to *PQ*



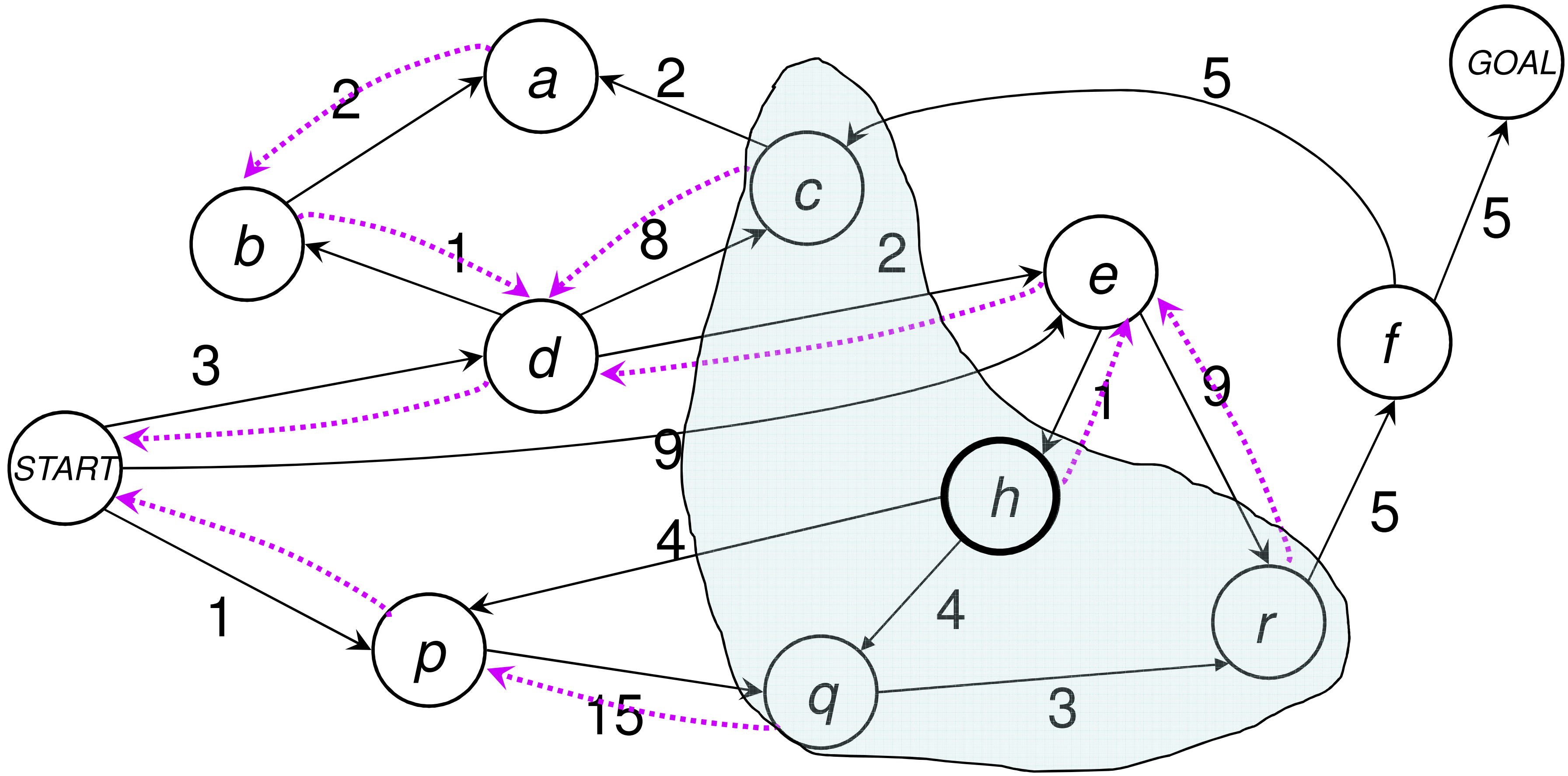
$PQ = \{(e,5)\ (a,6)\ (c,11)\ (q,16)\}$

1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* to *PQ*



$PQ = \{(a,6)\ (h,6)\ (c,11)\ (r,14)\ (q,16)\}$

1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* to *PQ*



$PQ = \{(h,6)\ (c,11)\ (r,14)\ (q,16)\}$

1. Pop the state *s* with the lowest path cost from *PQ*
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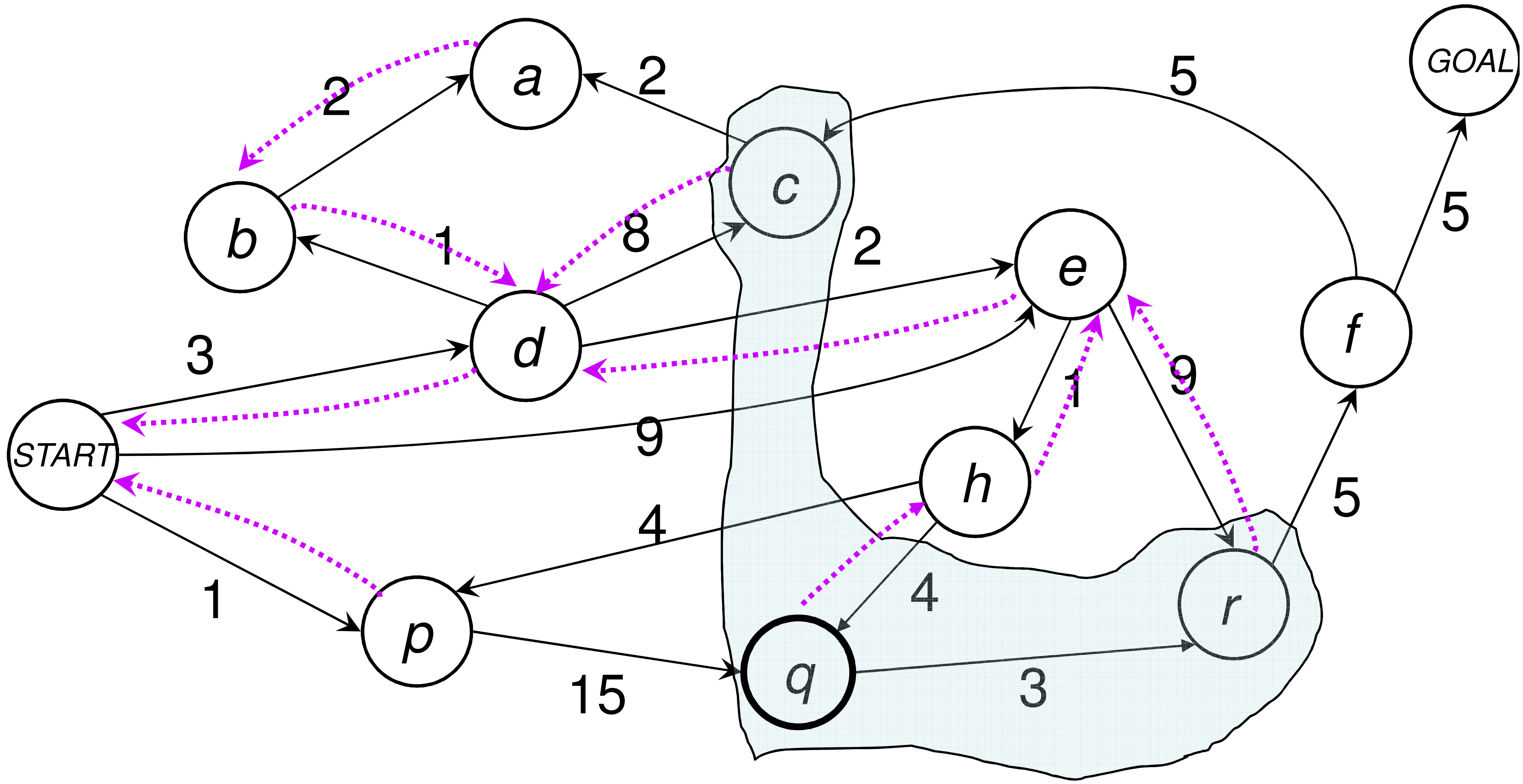
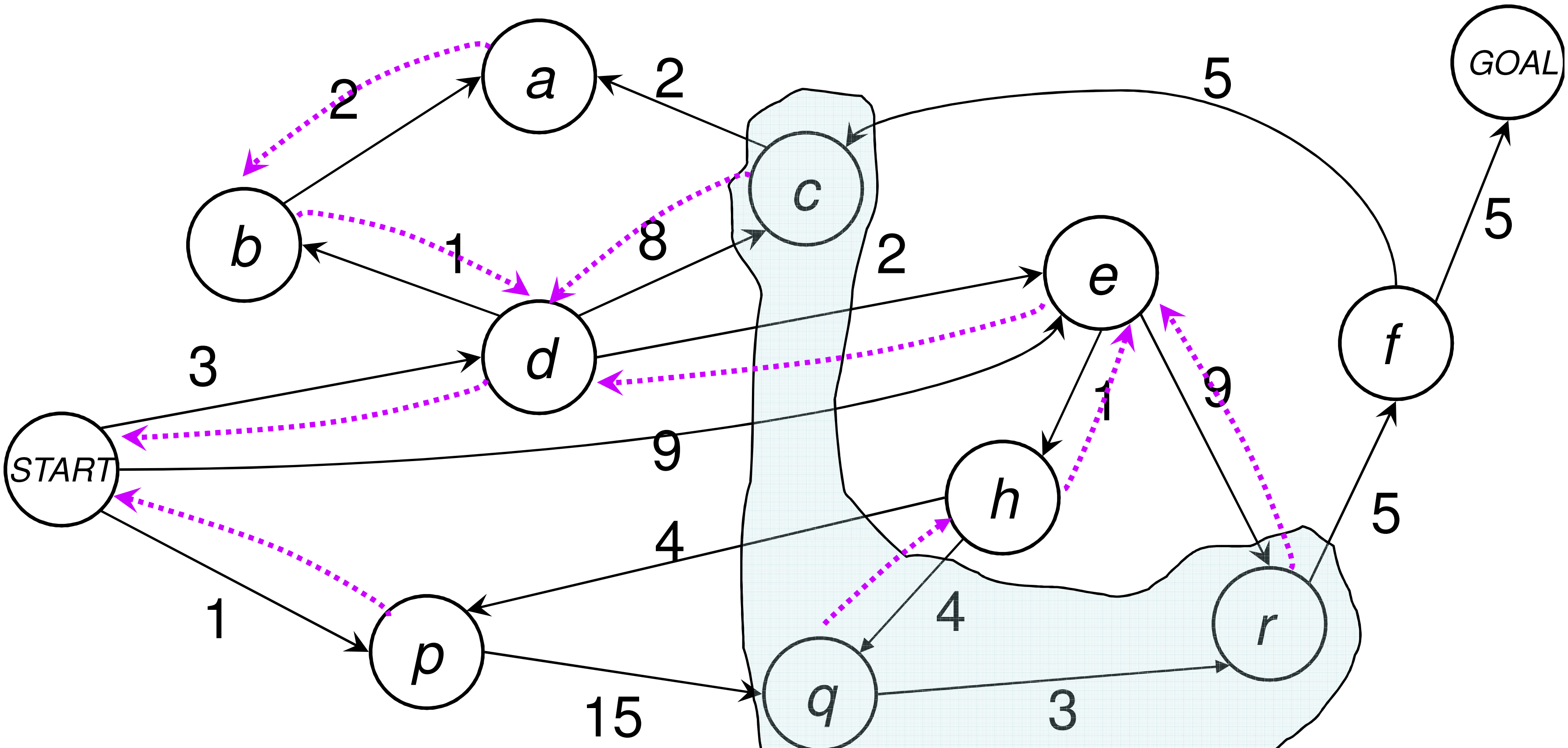


$PQ = \{(q,10)\ (c,11)\ (r,14)\}$

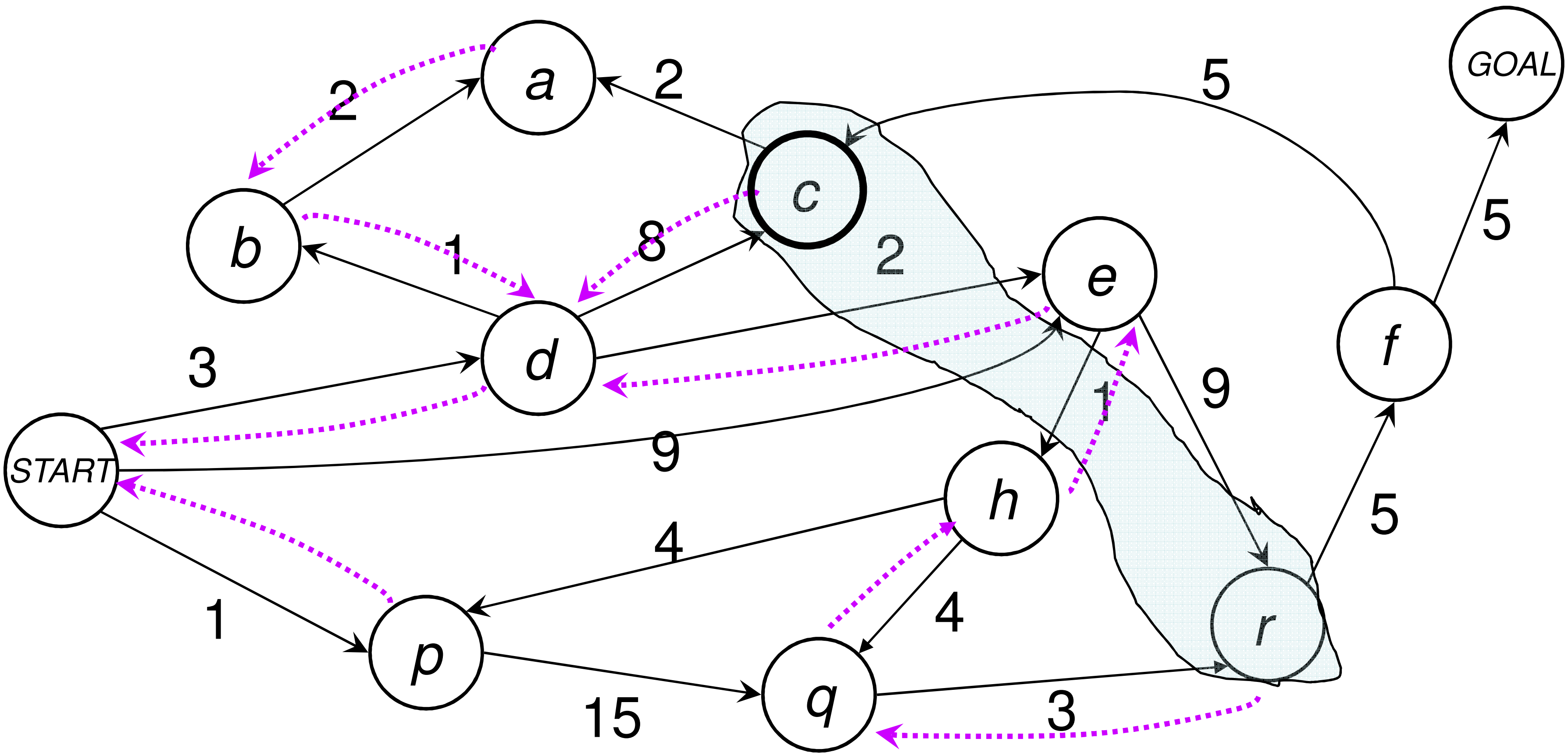
1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* to *PQ*



PQ = {(q,10) (c,11) (

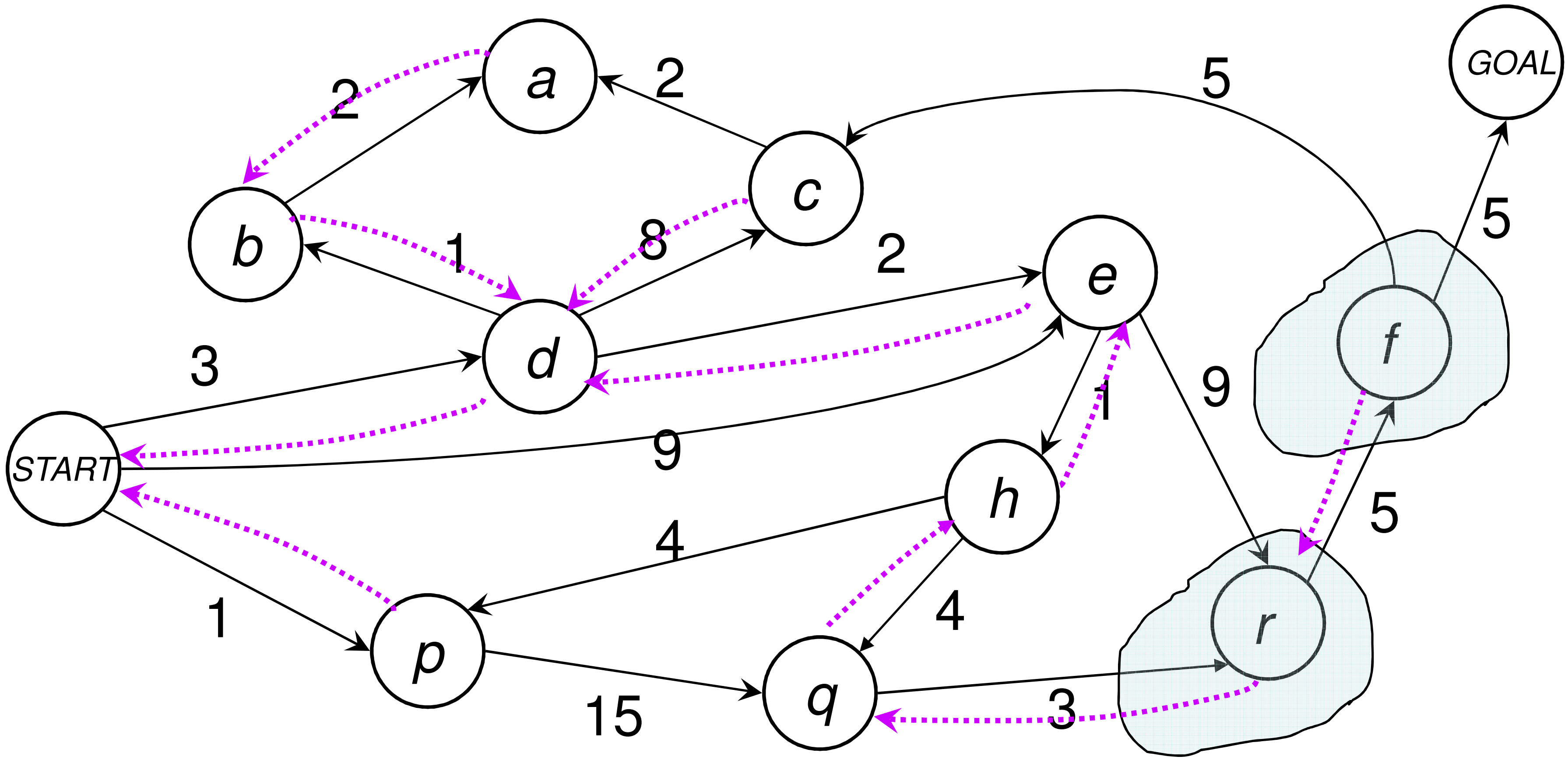
Important: We realized that going to ***q*** through ***h*** is cheaper than going through ***p*** → the value of ***q*** is updated from **16** to **10** and it moves up in PQ

he
PQ
t to
f *s* to



PQ = {(c,11) (r,13)}

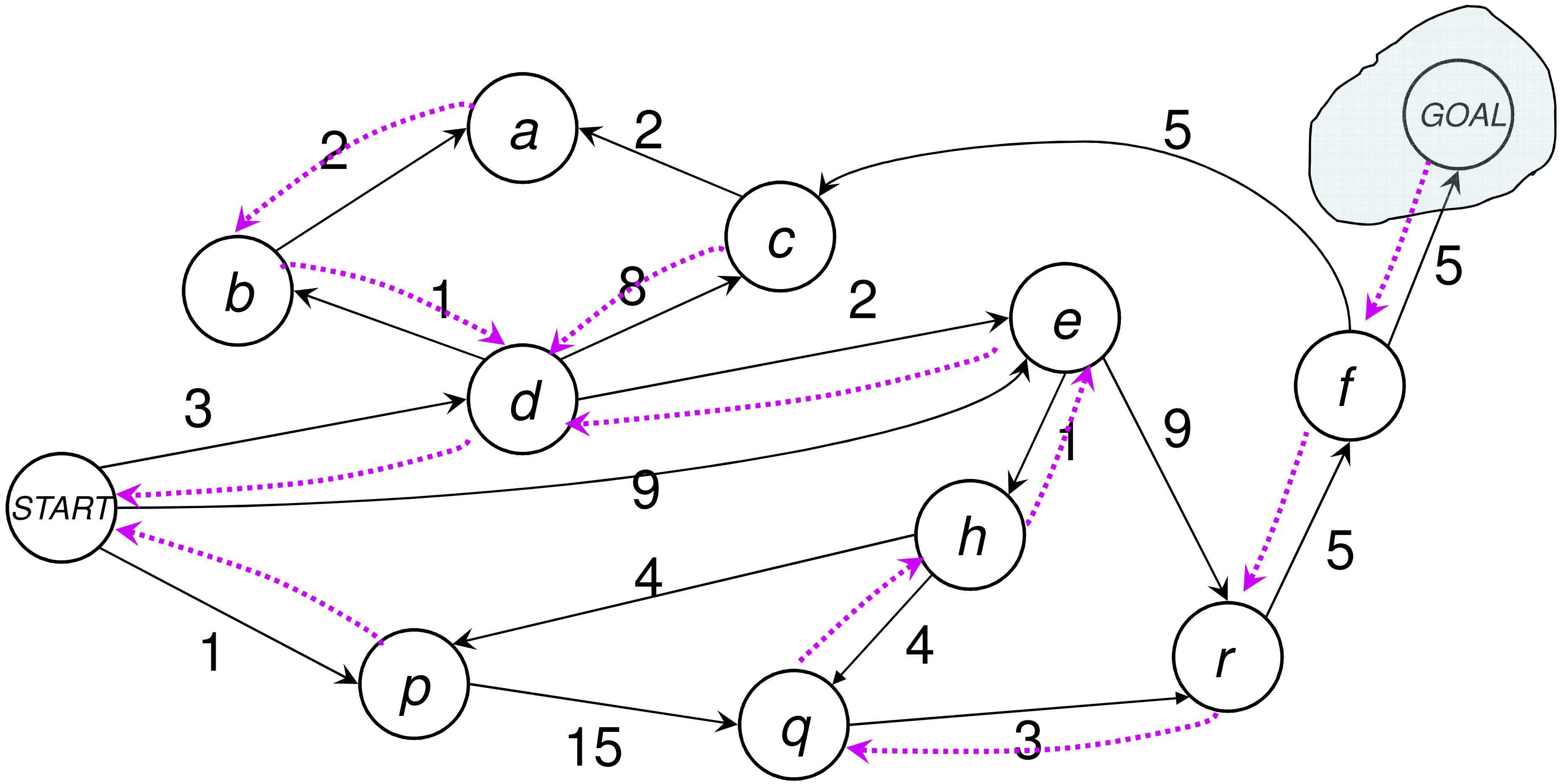
1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* to *PQ*



PQ = {(*r*,13)}

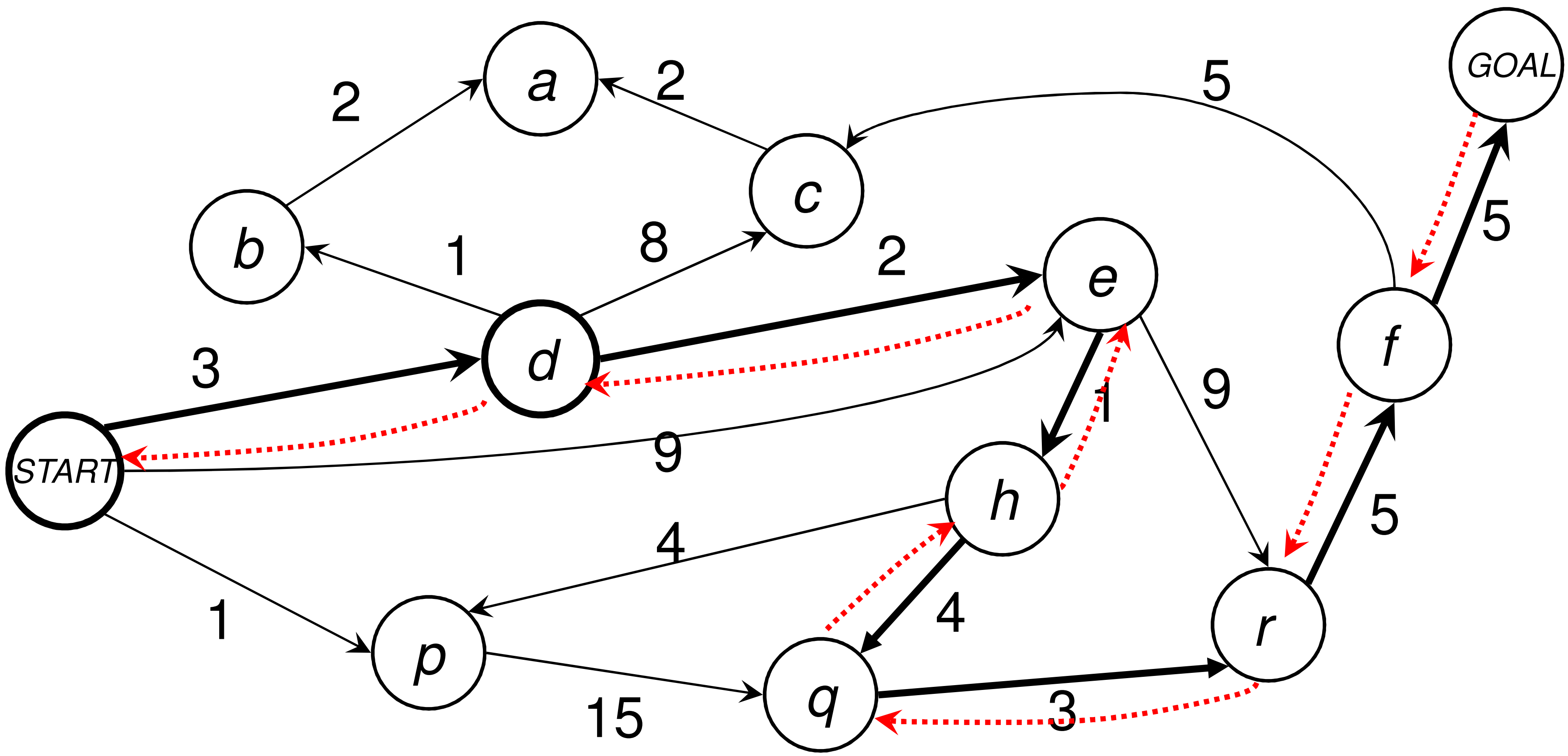
PQ = {(*f*,18)}

1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* to *PQ*



$PQ = \{(GOAL, 23)\}$

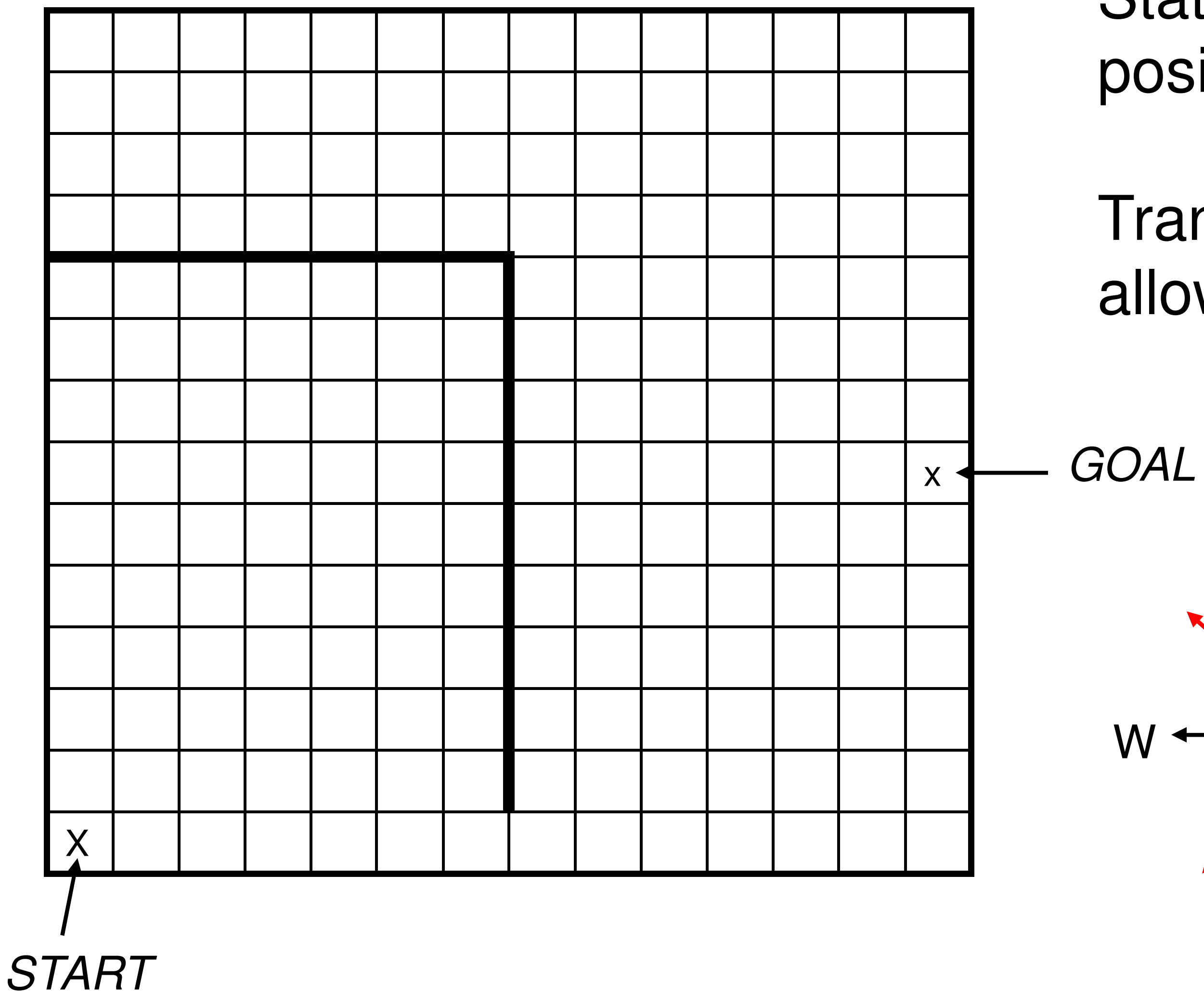
1. Pop the state *s* with the lowest path cost from *PQ*
2. Evaluate the path cost to all the successors of *s*
3. Add the successors of *s* to *PQ*



Final path: {*START*, *d*, *e*, *h*, *q*, *r*, *f*, *GOAL*}

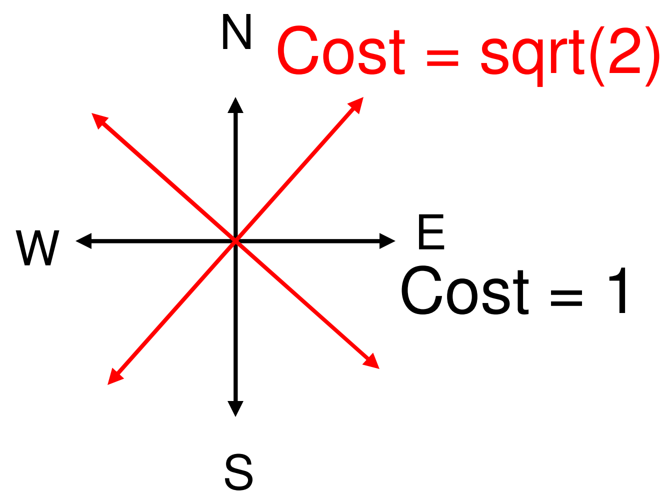
- This path is optimal in total cost even though it has more transitions than the one found by BFS
- What should be the stopping condition?
- Under what conditions is UCS complete/optimal?

Example: Robot Navigation



States =
positions in the map

Transitions =
allowed motions



Navigation: Going from point START to point GOAL given a (deterministic) map

Complexity

- N = Total number of states
- B = Average number of successors (branching factor)
- L = Length for start to goal with smallest number of steps
- Q = Average size of the priority queue

Algorithm		Complete	Optimal	Time	Space
BFS	Breadth First Search				
BIBFS	Bi-directional Breadth First Search				
UCS	Uniform Cost Search				

Complexity

- N = Total number of states
- B = Average number of successors (branching factor)
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Algorithm		Complete	Optimal	Time	Space
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UCS	Uniform Cost Search				

Complexity

- N = Total number of states
- B = Average number of successors (branching factor)
- L = Length for start to goal with smallest number of steps
- C = Cost of optimal path
- Q = Average size of the priority queue
- ε = average cost per link?

Algorithm		Complete	Optimal	Time	Space
BFS	Breadth First Search	Y	Y, If all trans. have same cost	$O(\min(N,B^L))$	$O(\min(N,B^L))$
BIBFS	Bi-directional Breadth First Search	Y	Y, If all trans. have same cost	$O(\min(N,2B^{L/2}))$	$O(\min(N,2B^{L/2}))$
UCS	Uniform Cost Search	Y, If cost > ε > 0	Y, If cost > 0	$O(\log(Q)*\min(N,B^{C/\varepsilon}))$	$O(\min(N,B^{C/\varepsilon}))$

Limitations of BFS

- Memory usage is $O(B^L)$ in general
- Limitation in many problems in which the states cannot be enumerated or stored explicitly, e.g., large branching factor
- Alternative: Find a search strategy that requires little storage for use in large problems

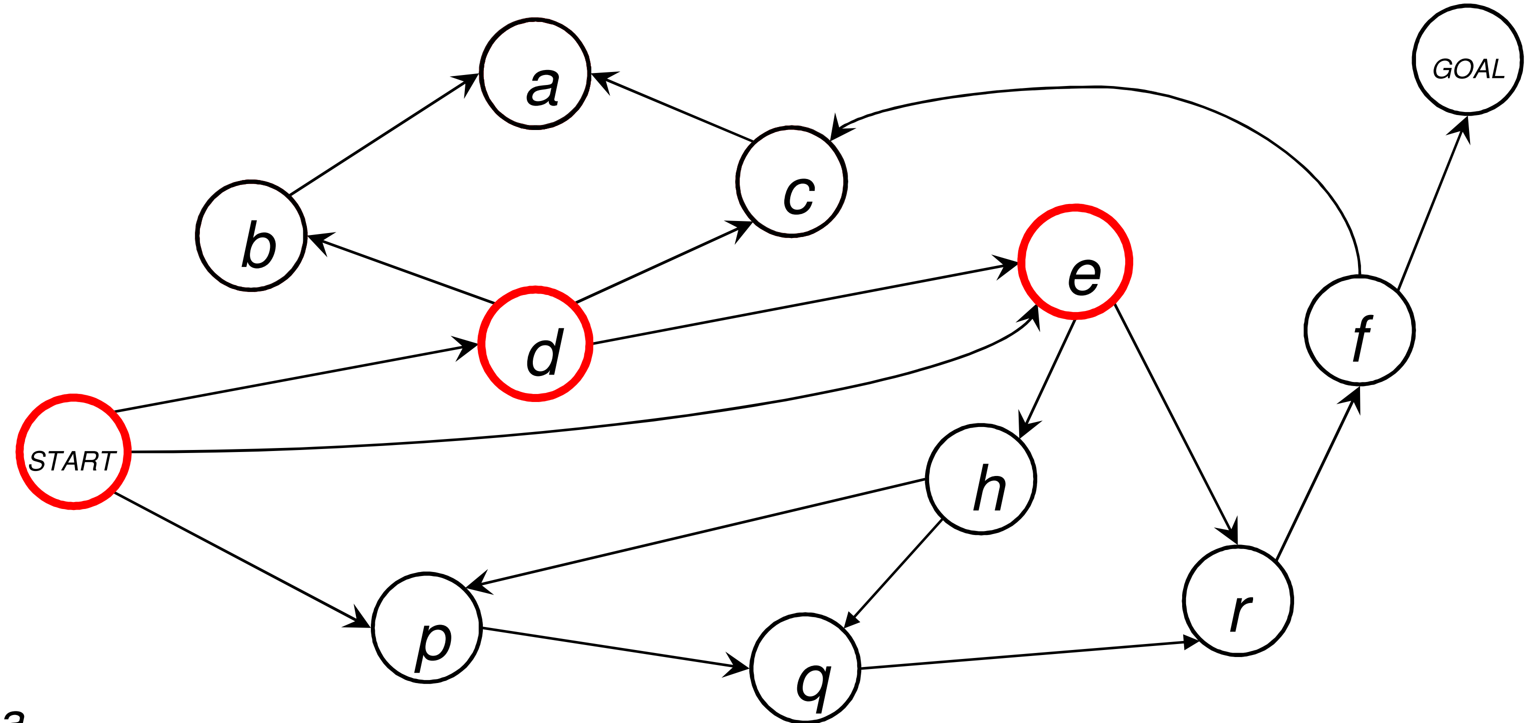
Philosophical Limitation

- We cannot shoot for perfection, we want good enough…

Depth First Search

"left first:"

START
START *d*
START *d b*
START *d b a*
START *d c*
START *d c a*
START *d e*
START *d e r*
START *d e r f*
START *d e r f c*
START *d e r f c a*
START *d e r f* GOAL



- General idea:
  - Expand the most recently expanded node if it has successors
  - Otherwise backup to the previous node on the current path

DFS Implementation

DFS (*s*)

if *s* = *GOAL*

return *SUCCESS*

else

For all *s*' in **succs**(*s*)

DFS (*s*')

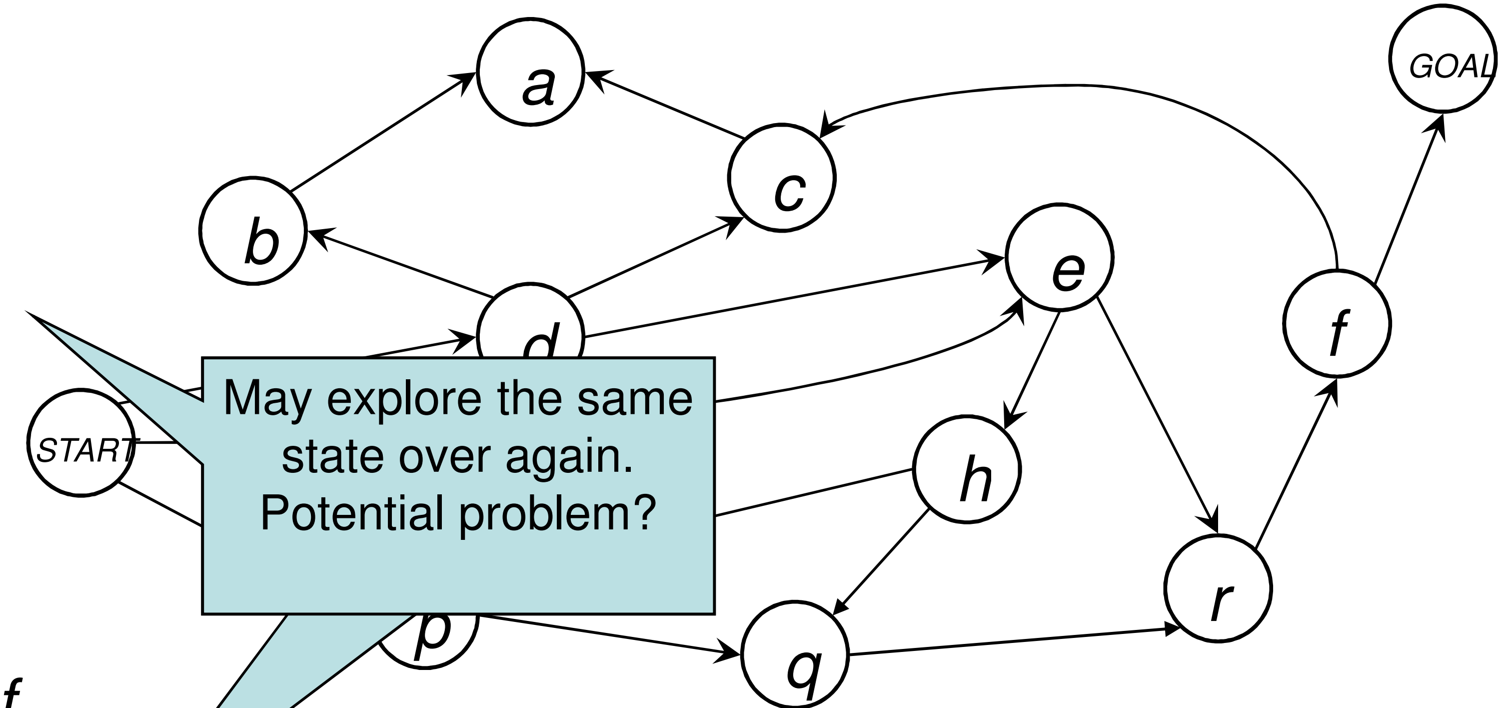
return *FAILURE*

In a recursive implementation, the program stack keeps track of the states in the current path

s is current state being expanded, starting with *START*

Depth First Search

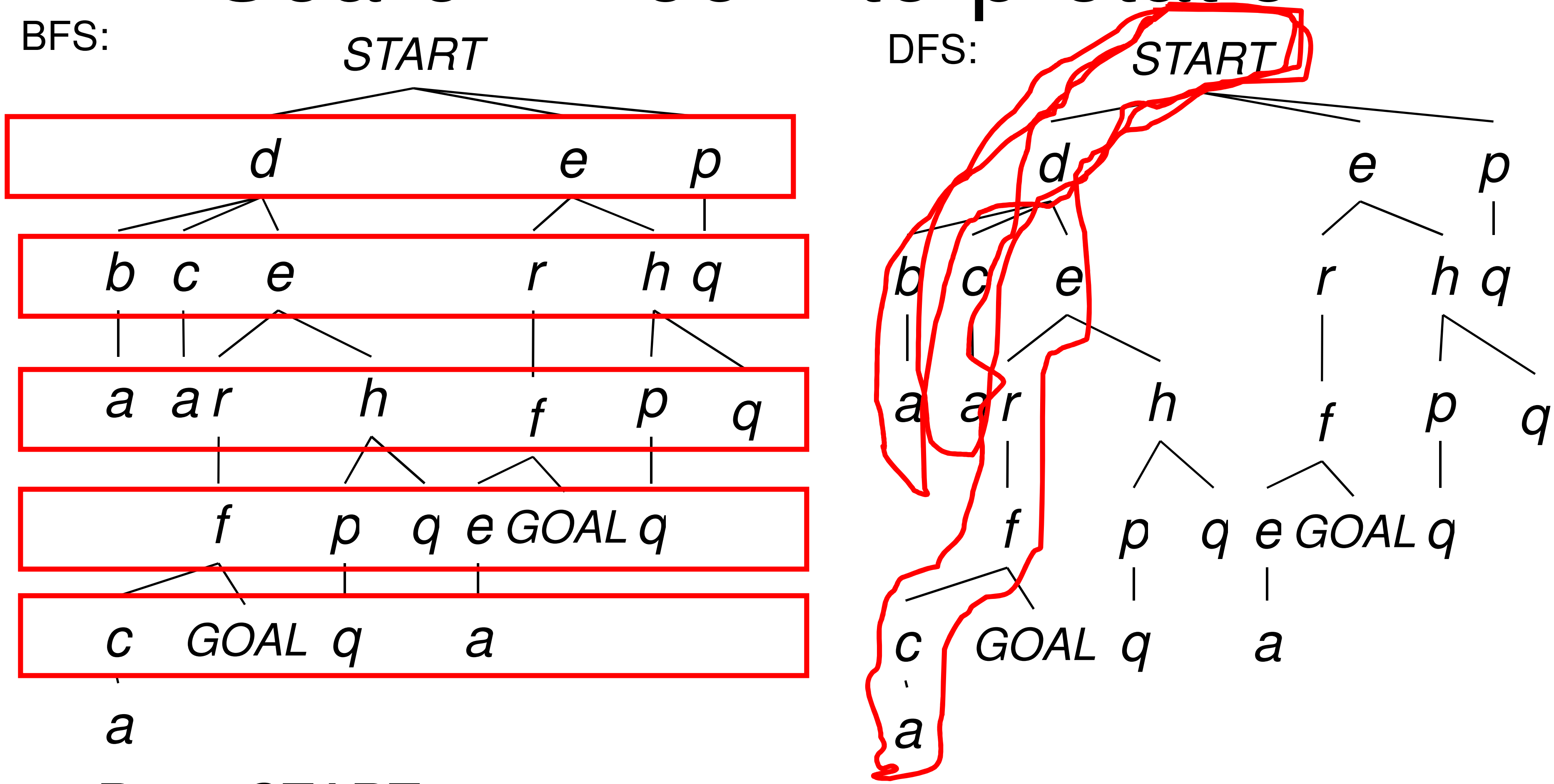
START
START d
START d b
START d b a
START d c
START d c a
START d e
START d e r
START d e r f
START d e r f c
START d e r f c a
START d e r f GOAL



May explore the same state over again. Potential problem?

Memory usage never exceeds maximum length of a path through the graph

Search Tree Interpretation



- Root: *START* state
- Children of node containing state *s*: All states in **succs**(*s*)
- In the worst case the entire tree is explored → O(B^{Lmax})
- Infinite branches if there are loops in the graph!

Complexity

- N = Total number of states
- B = Average number of successors (branching factor)
- L = Length for start to goal with smallest number of steps
- C = Cost of optimal path
- Q = Average size of the priority queue
- $Lmax$ = Length of longest path from *START* to any state

Algorithm		Complete	Optimal	Time	Space
BFS	Breadth First Search				
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UCS	Uniform Cost Search	Y (if cost > 0)	Y	$O(\log(Q)*\min(N,B^{C/\varepsilon}))$	$O(\min(N,B^{C/\varepsilon}))$
DFS	Depth First Search				

Complexity

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DFS	Depth First Search	Y	N	$O(B^{Lmax})$	$O(BL_{max})$

For graphs without cycles

Complexity

Is this a problem:

- *Lmax* = Length of longest path from *START* to any state

Algorithm		Complete	Optimal	Time	Space
BFS	Breadth First Search	Y	Y, If all trans. have same cost	$O(\min(N,B^{L}))$	$O(\min(N,B^{L}))$
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DFS	Depth First Search	Y	N	$O(B^{Lmax})$	$O(BL_{max})$

For graphs without cycles

DFS Limitation 1

- Need to prevent DFS from looping
- Avoid visiting the same states repeatedly

Because B^d may be much larger than the number of states d steps away from the start

- PC-DFS (Path Checking DFS):
  - Don't use a state that is already in the current path
- MEMDFS (Memorizing DFS):
  - Keep track of all the states expanded so far. Do not expand any state twice
- Comparison PC-DFS vs. MEMDFS?

Complexity

- *N* = Total number of states
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	Algorithm	Complete	Optimal	Time	Space
BFS	Breadth First Search				
BIBFS	Bi- Direction. BFS				
UCS	Uniform Cost Search				
PCDFS	Path Check DFS				
MEMDFS	Memorizing DFS				

Complexity

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PCDFS	Path Check DFS	Y	N	$O(B^{Lmax})$	$O(BL_{max})$
MEMDFS	Memorizing DFS	Y	N	$O(Min(N,B^{Lmax}))$	$O(Min(N,B^{Lmax}))$

DFS Limitation 2

- Need to make DFS optimal

“Depth-Limited Search”

- IDS (Iterative Deepening Search):
  - Run DFS by searching only path of length 1 (DFS stops if length of path is greater than 1)
  - If that doesn’t find a solution, try again by running DFS on paths of length 2 or less
  - If that doesn’t find a solution, try again by running DFS on paths of length 3 or less
  - ………..
  - Continue until a solution is found

Iterative Deepening Search

- Sounds horrible: We need to run DFS many times
- Actually not a problem:

$$O(LB^1+(L-1)B^2+\ldots+B^L) = O(B^L)$$

Nodes generated at depth 1

Nodes generated at depth 2

Nodes generated at depth L

- Compare B^L and B^{Lmax}
- Optimal if transition costs are equal

Iterative Deepening Search (DFID)

- Memory usage same as DFS
- Computation cost comparable to BFS even with repeated searches, especially for large *B*.
- Example:
  - *B*=10, *L*=5
  - BFS: 111,111 expansions
  - IDS: 123,456 expansions

Complexity

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IDS	Iterative Deepening				

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PCDFS	Path Check DFS	Y	N	O(B^{Lmax})	O(BL_{max})
MEMDFS	Memorizing DFS	Y	N	O(Min(N,B^{Lmax}))	O(Min(N,B^{Lmax}))
IDS	Iterative Deepening	Y	Y, If all trans. have same cost	O(B^L)	O(BL)

Summary

- Basic search techniques: BFS, UCS, PCDFS, MEMDFS, DFID
- Property of search algorithms: Completeness, optimality, time and space complexity
- Iterative deepening and bidirectional search ideas
- Trade-offs between the different techniques and when they might be used

Some Challenges

- Driving directions
- Robot navigation in Wean Hall
- Adversarial games
  - Tic Tac Toe
  - Chess