AI
Decision Tree Learning – Part I

Illah Nourbakhsh's version

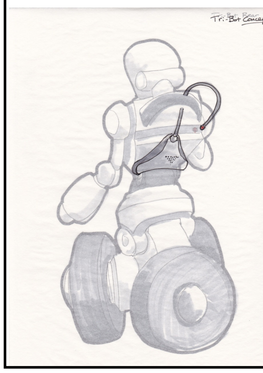
Chapter 18, Russell and Norvig

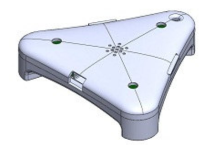
Thanks to all past instructors

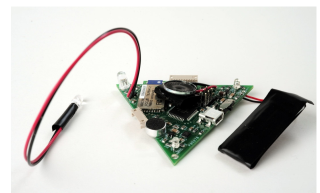
Carnegie Mellon

brainlinksystem.com

$25+ / hr







Outline

- Learning and philosophy
- Induction versus Deduction
- Decision trees: introduction

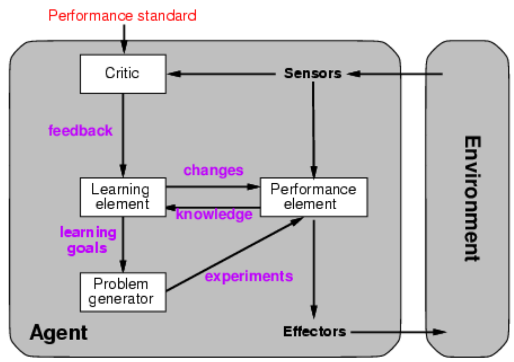
Learning

- How do we define learning?
- Gathering more knowledge
  - "Knowing more than was known before learning"
- Learning "substitutes" the need to model *a priori*
- Experience, feedback, refinement
- Learning modifies the agent's decision mechanisms to improve performance

Learning

- Declarative versus Procedural Knowledge
- Explicit versus Implicit Knowledge
- Induction versus Deduction

Learning agents



Learning "Element"

- A bit of "magic":
  - Which **components** of the performance element are to be learned
  - What **feedback** is available to learn these components
  - What **representation** is used for the components
- Type of feedback:
  - Supervised learning: correct labels/answers
  - Unsupervised learning: {correct labels/answers} missing
  - Reinforcement learning: occasional rewards

Inductive Learning

- Simplest form: learn a function from **examples**

f is the target function

An example is a pair (*x*, *f(x)*) – *supervised learning*

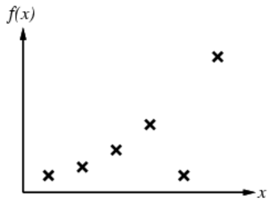
Problem: find a hypothesis *h*

such that $h \approx f$

given a training set of examples

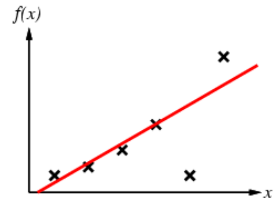
Inductive Learning Method

- Construct/adjust *h* to agree with *f* on training set
- *h* is consistent if it agrees with *f* on all examples
- E.g., curve fitting:



Inductive Learning Method

- Construct/adjust *h* to agree with *f* on training set
- *h* is consistent if it agrees with *f* on all examples
- E.g., curve fitting (good idea? Bad idea?):



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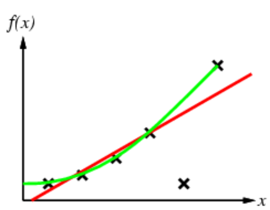

Inductive Learning Method

- Construct/adjust *h* to agree with *f* on training set
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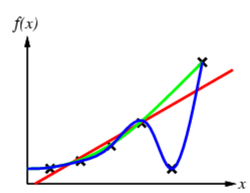
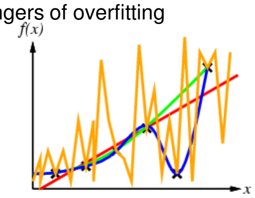


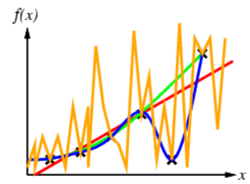
Inductive Learning Method

- Construct/adjust *h* to agree with *f* on training set
- *h* is consistent if it agrees with *f* on all examples
- The dangers of overfitting



Inductive Learning Method - Bias

- Hypothesis "form" – **bias** on the learning outcome
- Occam's razor: prefer the simplest hypothesis consistent with data



Attribute-Based Data Sets

- Examples described by attribute values (Boolean, discrete, continuous)
- Function is class, wait/not wait for table at restaurant

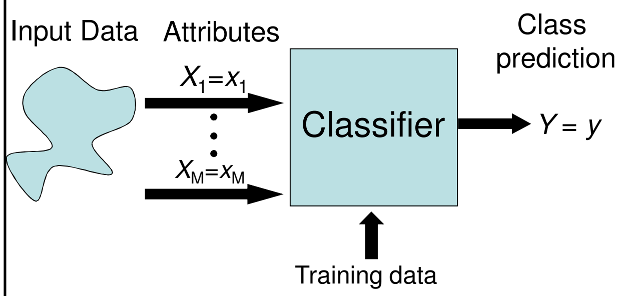
Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

- Classification of examples is positive (T) or negative (F)

Attribute-Based Data Set

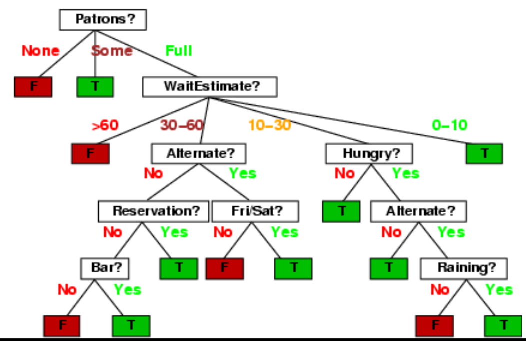
- *Type*: drama, comedy, thriller
- *Company*: MGM, Columbia
- *Director*: Bergman, Spielberg, Hitchcock
- *Mood*: stressed, relaxed, normal

Movie	Type	Company	Director	Mood	Likes-movie?
m_1	thriller	MGM	Bergman	normal	No
m_2	comedy	Columbia	Spielberg	stressed	Yes
m_3	comedy	MGM	Spielberg	relaxed	No
m_4	thriller	MGM	Bergman	relaxed	No
m_5	comedy	MGM	Hitchcock	normal	Yes
m_6	drama	Columbia	Bergman	relaxed	Yes
m_7	drama	Columbia	Bergman	normal	No
m_8	drama	MGM	Spielberg	stressed	No
m_9	drama	MGM	Hitchcock	normal	Yes
m_{10}	comedy	Columbia	Spielberg	relaxed	No
m_{11}	thriller	MGM	Spielberg	normal	No
m_{12}	thriller	Columbia	Hitchcock	relaxed	No



Data Representation - Decision Trees

- One possible representation for hypotheses
- E.g., here is the "true" tree for deciding whether to wait:



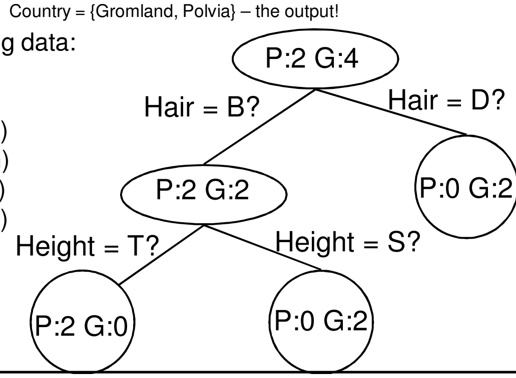
Expressiveness

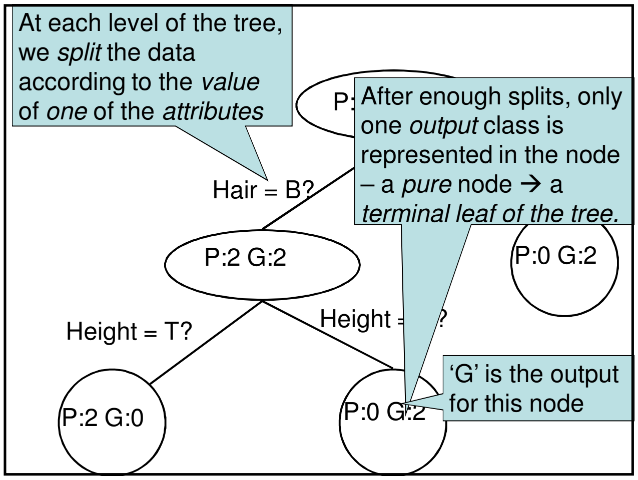
- Decision trees can express any function of the input attributes
  - E.g., for Boolean functions, truth table row → path to leaf
- Trivially, there is a consistent decision tree for any training set with one path to leaf for each example (unless *f* nondeterministic in *x*)
  - But… it probably won't generalize to new examples – goal of learning…
- Goal: Find more "compact" decision trees

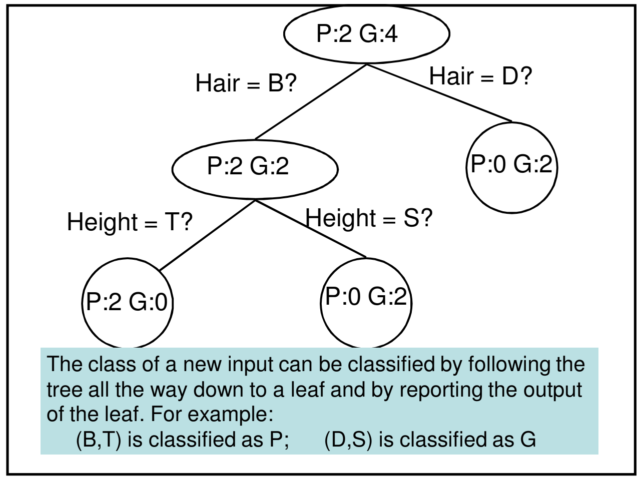
Building a Decision Tree

- Three variables:
  - Hair = {blond, dark}
  - Height = {tall,short}
  - Country = {Gromland, Polvia} – the output!

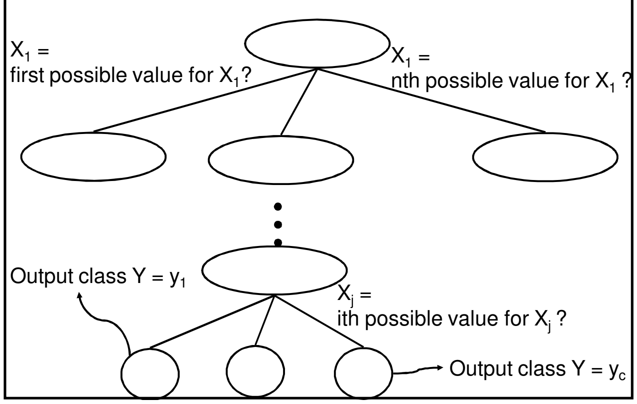
Training data:
(B,T,P)
(B,T,P)
(B,S,G)
(D,S,G)
(D,T,G)
(B,S,G)







General Decision Tree



Basic Questions

- How **to choose the attribute to split** on at each level of the tree?
- When **to stop splitting**? When should a node be declared a leaf?
- If a leaf node is impure, how should the **class label be assigned**?
- If the tree is too large, how can it be **pruned**?

Set of Training Examples

- *Type*: drama, comedy, thriller
- *Company*: MGM, Columbia
- *Director*: Bergman, Spielberg, Hitchcock
- *Mood*: stressed, relaxed, normal

Movie	Type	Company	Director	Mood	Likes-movie?
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m_7	drama	Columbia	Bergman	normal	No
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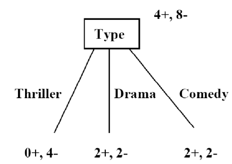
m_{10}	comedy	Columbia	Spielberg	relaxed	No
m_{11}	thriller	MGM	Spielberg	normal	No
m_{12}	thriller	Columbia	Hitchcock	relaxed	No

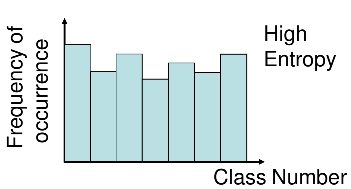
Choosing the "Best" Attribute

- Ideal attribute – partitions examples into all positive or all negative (or from the same class in each partition).
- Attribute that results in higher "discrimination."

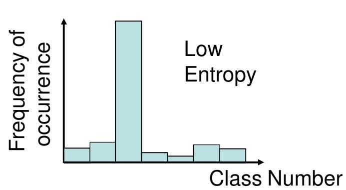
How *good* is the attribute "Type" of movie?



Entropy *(Shannon and Weaver 1949)*



The entropy captures the degree of "purity" of the data distribution



Entropy *(Shannon and Weaver 1949)*

- In general, entropy H is the average number of bits necessary to encode *n* values:

$$\boxed{H = -\sum_{i=1}^{n} - P_i \log_2 P_i}$$

- P_i = probability of occurrence of value *i*
  - High entropy → All the classes are (nearly) equally likely; low information
  - Low entropy → A few classes are likely; most of the classes are rarely observed; high level of information gain

Entropy for Attribute Choice

- Measure of *information* provided by the attribute
- **Entropy** of a set of examples S as the information content of S.

$$\text{Entropy(S)} = \sum_{i=1}^{c} -p_i \log_2 p_i$$

where $p_i = \frac{|S_i|}{|S|}$

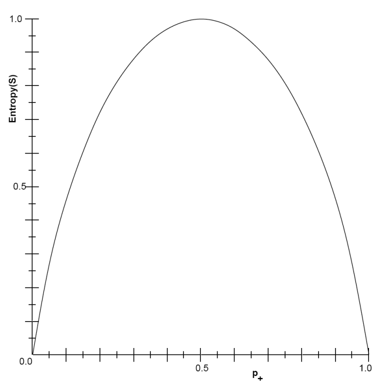
- c classes, Si size of the data set for class i

Entropy

- Unit: 1 bit of information =
  - the information content of the actual answer when there are two possible answers equally probable.

$$E(S) = -\tfrac{1}{2}\log_2 \tfrac{1}{2} - \tfrac{1}{2}\log_2 \tfrac{1}{2} = 1$$

Entropy Relative – Boolean Classification



Entropy – Example

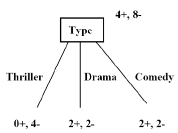
- $|S| = 12$, $c = 2(+,-)$, $|S_+| = 4$, $|S_-| = 8$

$$\begin{aligned} E(S) &= -\tfrac{4}{12}\log_2 \tfrac{4}{12} - \tfrac{8}{12}\log_2 \tfrac{8}{12} \\ &= 0.918 \end{aligned}$$

Choosing the "Best" Attribute

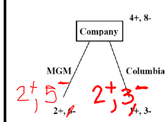
- Attribute with **highest information gain** – expected reduction in entropy of the set S if partitioned according to attribute A.

$$\text{Gain}(S,A) = \text{Entropy}(S) - \sum_{i=1}^{\text{values}(A)} \frac{|S_{v_i}|}{|S|}\text{Entropy}(S_{v_i})$$

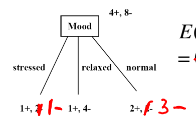


$$\begin{aligned}\text{Gain}(S,\text{Type}) = E(4^+,8^-) & - \tfrac{4}{12}E(0^+,4^-) \\ & - \tfrac{4}{12}E(2^+,2^-) \\ & - \tfrac{4}{12}E(2^+,2^-) \\ = 0.252 &\end{aligned}$$

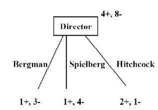
Other Attributes



$$E(4^+,8^-) - \tfrac{\cancel{8}\,7}{12}E(2^+,\cancel{6^-}\,5^-) - \tfrac{\cancel{4}\,5}{12}E(\cancel{1^+}\,2^+,3^-)$$
$$= \cancel{0.107}$$
$$= 0.0102$$

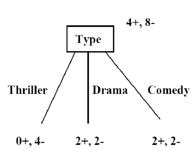


$$E(4^+,8^-) - \tfrac{\cancel{3}\,2}{12}E(1^+,\cancel{2^-}\,1^-) - \tfrac{5}{12}E(1^+,4^-) - \tfrac{\cancel{4}\,5}{12}E(2^+,\cancel{1^-}\,3^-)$$
$$= \cancel{0.055}$$
$$= 0.0462$$



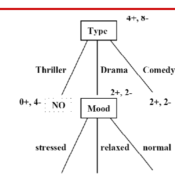
$$E(4^+,8^-) - \tfrac{4}{12}E(1^+,3^-) - \tfrac{5}{12}E(1^+,4^-) - \tfrac{3}{12}E(2^+,1^-)$$
$$= 0.118$$

Commitment to One Hypothesis



What is the next *best* attribute? Recursive.

Split Data and Continue

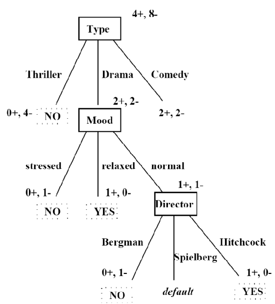


$$\text{Gain}(S_d,\text{Mood}) = E(2^+,2^-) - \tfrac{1}{4}E(0^+,1^-) - \tfrac{1}{4}E(1^+,0^-) - \tfrac{2}{4}E(1^+,1^-)$$
$$= 0.5$$

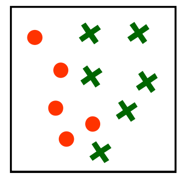
$$\text{Gain}(S_d,\text{Company}) = E(2^+,2^-) - \tfrac{2}{4}E(1^+,1^-) - \tfrac{2}{4}E(1^+,1^-)$$
$$= 0$$

$$\text{Gain}(S_d,\text{Director}) = E(2^+,2^-) - \tfrac{2}{4}E(1^+,1^-) - \tfrac{1}{4}E(0^+,1^-) - \tfrac{2}{4}E(1^+,0^-)$$
$$= 0.5$$

A Learned Tree

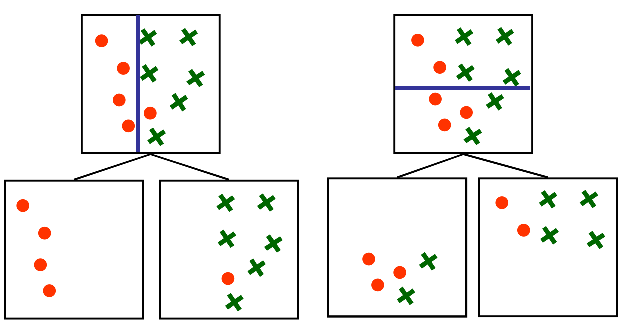


Continuous Case- How to Split?



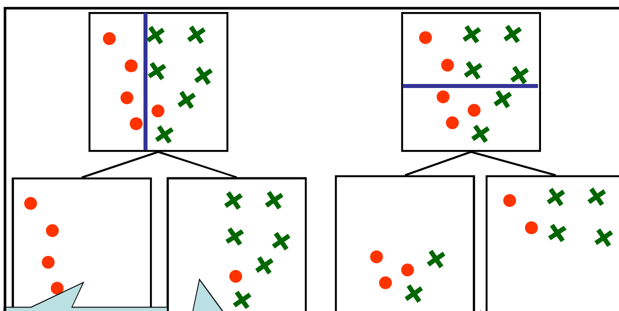
- Two classes (red circles/green crosses)
- Two attributes: X_1 and X_2
- 11 points in training data
- Idea → Construct a decision tree such that the leaf nodes predict correctly the class for all the training examples

Good and Bad Splits



Good

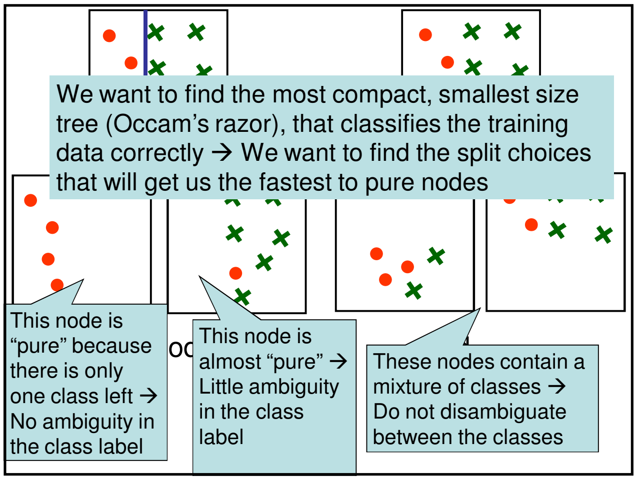
Bad

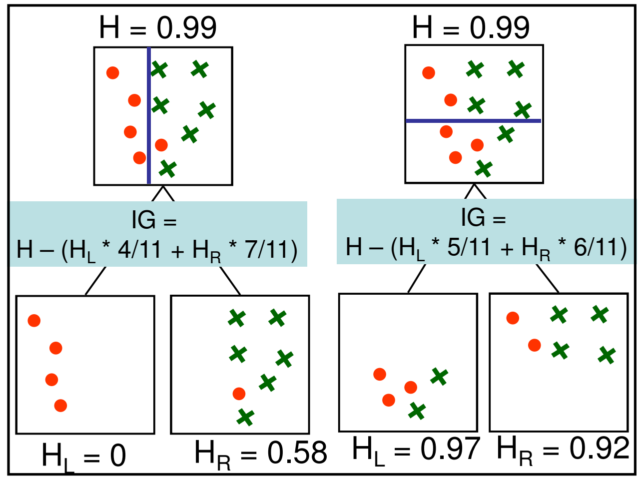


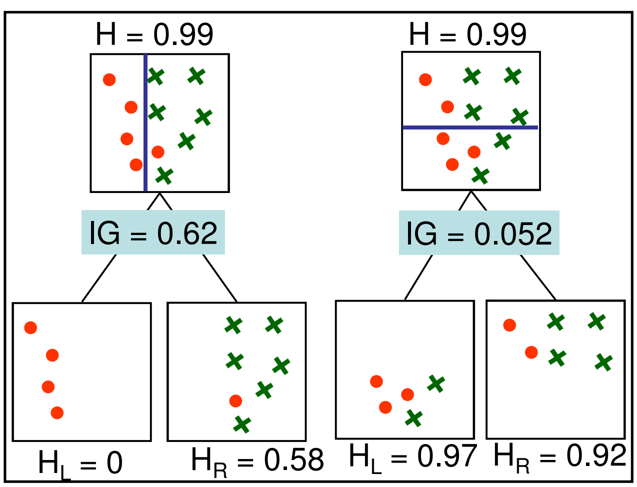
This node is "pure" because there is only one class left → No ambiguity in the class label

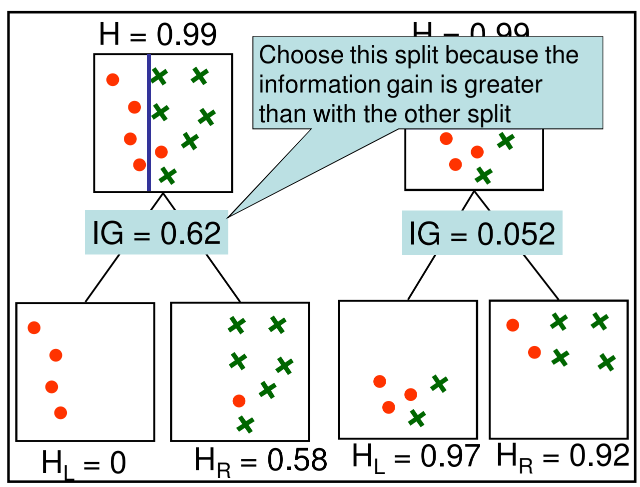
This node is almost "pure" → Little ambiguity in the class label

These nodes contain a mixture of classes → Do not disambiguate between the classes

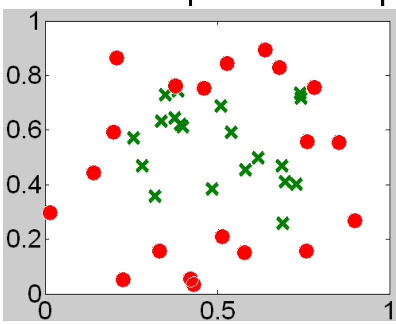

We want to find the most compact, smallest size tree (Occam's razor), that classifies the training data correctly → We want to find the split choices that will get us the fastest to pure nodes
This node is "pure" because there is only one class left → No ambiguity in the class label
This node is almost "pure" → Little ambiguity in the class label
These nodes contain a mixture of classes → Do not disambiguate between the classes


H = 0.99
H = 0.99
IG = H – (H_L * 4/11 + H_R * 7/11)
IG = H – (H_L * 5/11 + H_R * 6/11)
H_L = 0
H_R = 0.58
H_L = 0.97
H_R = 0.92


H = 0.99
H = 0.99
IG = 0.62
IG = 0.052
H_L = 0
H_R = 0.58
H_L = 0.97
H_R = 0.92


H = 0.99
Choose this split because the information gain is greater than with the other split
IG = 0.62
IG = 0.052
H_L = 0
H_R = 0.58
H_L = 0.97
H_R = 0.92

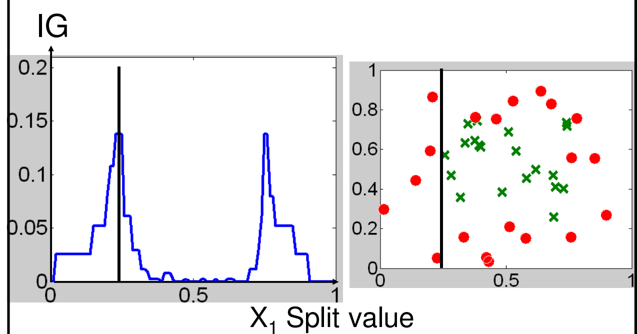
More Complete Example



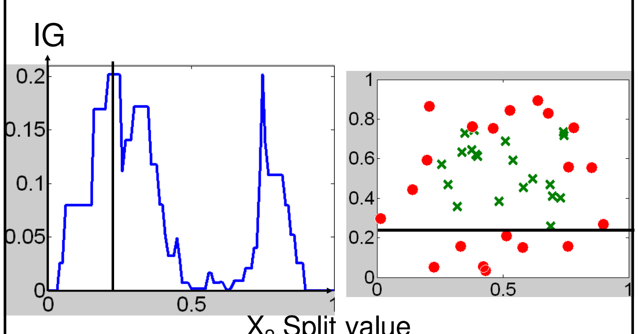
= 20 training examples from class A

= 20 training examples from class B

Attributes = X_1 and X_2 coordinates



Best split value (max Information Gain) for X_1 attribute: 0.24 with IG = 0.138

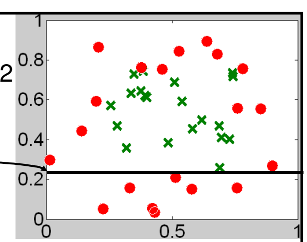


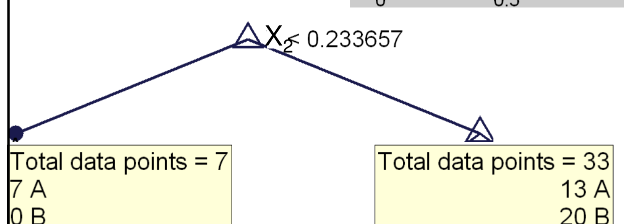
Best split value (max Information Gain) for X_2 attribute: 0.234 with IG = 0.202

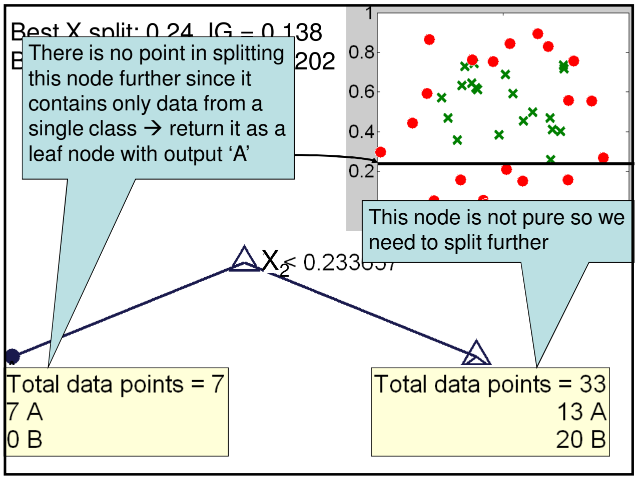
Best X_1 split: 0.24, IG = 0.138

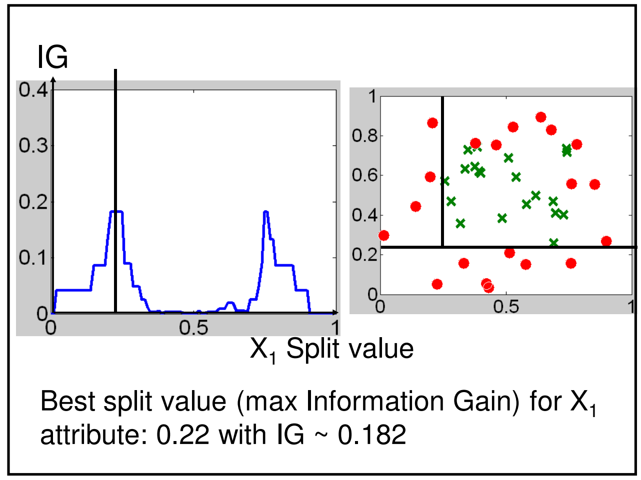
Best X_2 split: 0.234, IG = 0.202

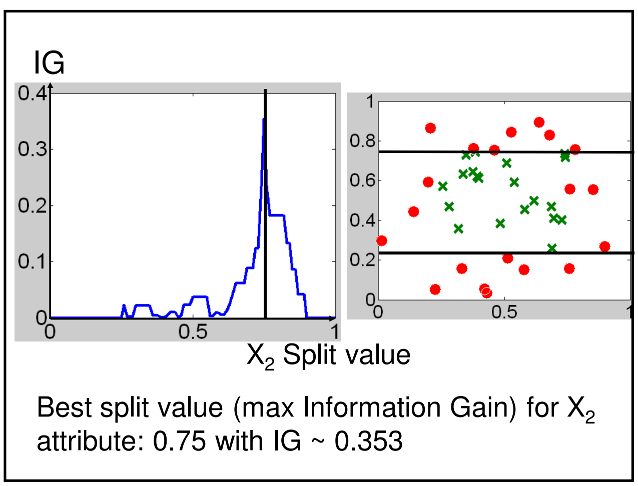
Split on X_2 with 0.234

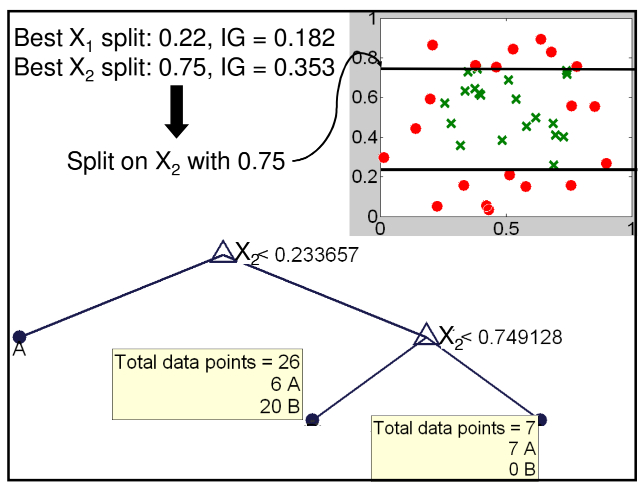


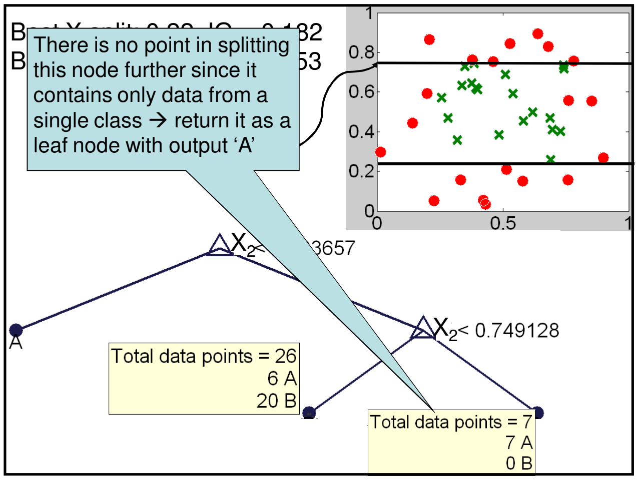


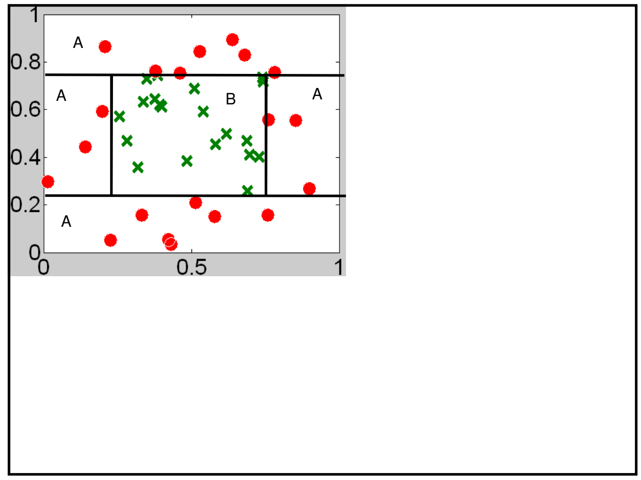

There is no point in splitting this node further since it contains only data from a single class → return it as a leaf node with output 'A'
This node is not pure so we need to split further
Total data points = 7
7 A
0 B
Total data points = 33
13 A
20 B

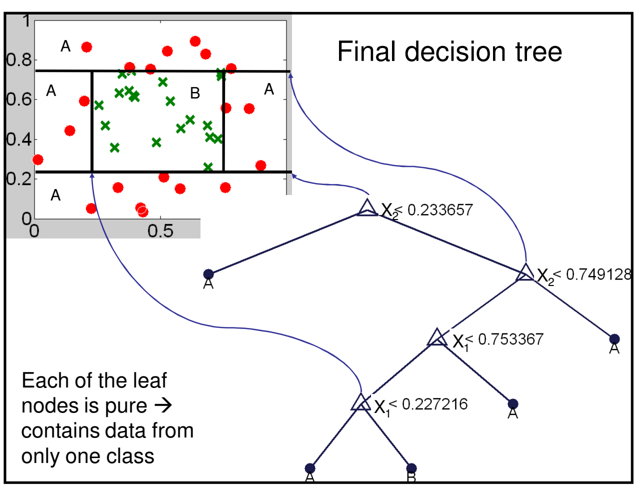

IG
X_1 Split value
Best split value (max Information Gain) for X_1 attribute: 0.22 with IG ~ 0.182

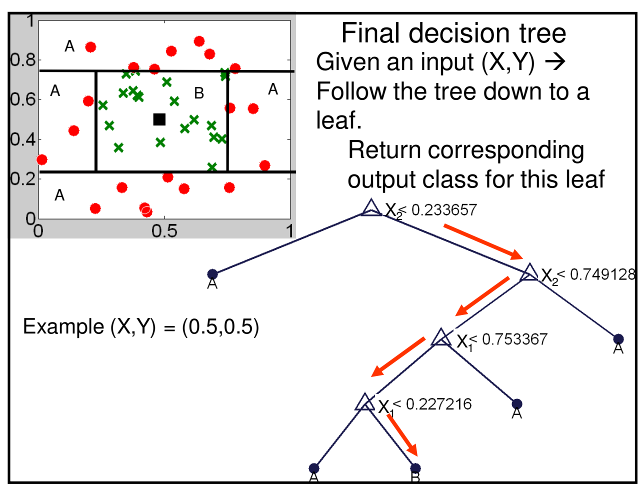

IG
X_2 Split value
Best split value (max Information Gain) for X_2 attribute: 0.75 with IG ~ 0.353


Best X_1 split: 0.22, IG = 0.182
Best X_2 split: 0.75, IG = 0.353
Split on X_2 with 0.75
X_2 < 0.233657
X_2 < 0.749128
A
Total data points = 26
6 A
20 B
Total data points = 7
7 A
0 B


There is no point in splitting this node further since it contains only data from a single class → return it as a leaf node with output 'A'
Total data points = 26
6 A
20 B
Total data points = 7
7 A
0 B
X2< 0.749128
A


A
A
B
A
A


Final decision tree
X2< 0.233657
X2< 0.749128
X1< 0.753367
X1< 0.227216
Each of the leaf nodes is pure → contains data from only one class


Final decision tree
Given an input (X,Y) →
Follow the tree down to a leaf.
Return corresponding output class for this leaf
Example (X,Y) = (0.5,0.5)
X2< 0.233657
X2< 0.749128
X1< 0.753367
X1< 0.227216

Induction of Decision Trees

ID3 (*examples*, *attributes*)
if there are no *examples*
  then return *default*
  else
    if all *examples* are members of the same class
      then return the class
      else
        if there are no *attributes*
          then return Most_Common_Class (*examples*)
          else
            best_attribute ← **Choose_Best** (*examples*, *attributes*)
            root ← Create_Node_Test (*best_attribute*)
            for each value v_i of *best_attribute*
              $examples_{v_i}$ ← subset of *examples* with *best_attribute* = v_i
              subtree ← **ID3**($examples_{v_i}$, *attributes* - *best_attribute*)
              set *subtree* as a child of *root* with label v_i
return *root*

Basic Questions

- How to choose the attribute/value to split on at each level of the tree?
- When to stop splitting? When should a node be declared a leaf?
- If a leaf node is impure, how should the class label be assigned?
- If the tree is too large, how can it be pruned?

Comments on Tree Termination

- Zero entropy in data set, perfect classification
  - Type = thriller, 0+ 4-
  - Type = drama, Mood = stressed, 0+ 1-
  - Type = drama, Mood = relaxed, 1+ 0-
  - Type = drama, Mood = normal, Director = Bergman, 0+ 1-
  - Type = drama, Mood = normal, Director = Hitchcock, 1+ 0-
- No examples availables
  - Type = drama, Mood = normal, Director = Spielberg
- Indistinguishable data
  - Type = comedy, attribute Company:
    - m2, comedy, Columbia, Yes, and m10, comedy, Columbia, No
    - m3, comedy, MGM, No, and m5, comedy, MGM, Yes
  - Type = comedy, attribute Director:
    - m2, Spielberg, Yes, and m3, Spielberg, No
    - m5, Hitchcock, Yes, and m10, Spielberg, No
  - (Type = comedy, attribute Mood, could have split?).

Errors

Split labeled data D into *training* and *validation* sets

- Training set error - fraction of training examples for which the decision tree disagrees with the true classification
- Validation set error - fraction of testing examples - from given labeled examples - for which the decision tree disagrees with the true classification

Overfitting

Tree too specialized – "perfectly" fit the training data.

A tree T *overfits* the training data, if there is an alternate T' such that

- for training set, error with T < error with T'
- for complete D, error with T > error with T'

- Node Pruning
- Rule Post-pruning
- Cross validation

Reduced-Error Pruning

- Remove subtree, make node a leaf node, assign the most common classification of the examples of that node.
- Check validation set error
- Continue pruning until error does not increase with respect to the error of the unpruned tree
- Rule post-pruning: represent tree as set of rules; remove rules independently; check validation set error; stop with same criteria

Other Issues

- Attributes with many values
  - Information gain may select it.
  - Consider splitting value and use the ratio between Gain and the splitting value
- Attributes with cost
  - Use other metrics
- Attributes with missing values
  - Infer most common value from examples at node

Summary

- Inductive learning – supervised learning – classification
- Decision trees represent hypotheses
- DT learning driven by information gain heuristic
- Recursive algorithm to build decision tree
- Next class:
  - More on continuous values
  - Missing, noisy attributes
  - Overfitting, pruning
  - Different attribute selection criteria