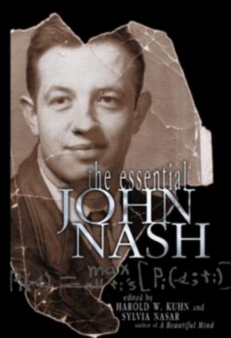
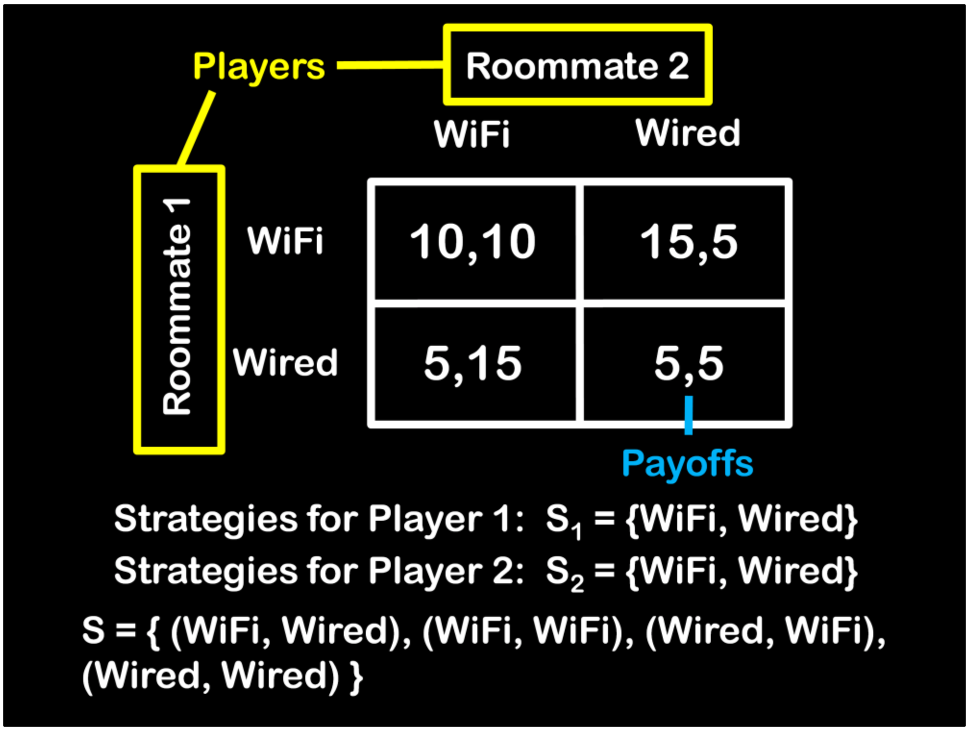
Preliminaries of Game Theory





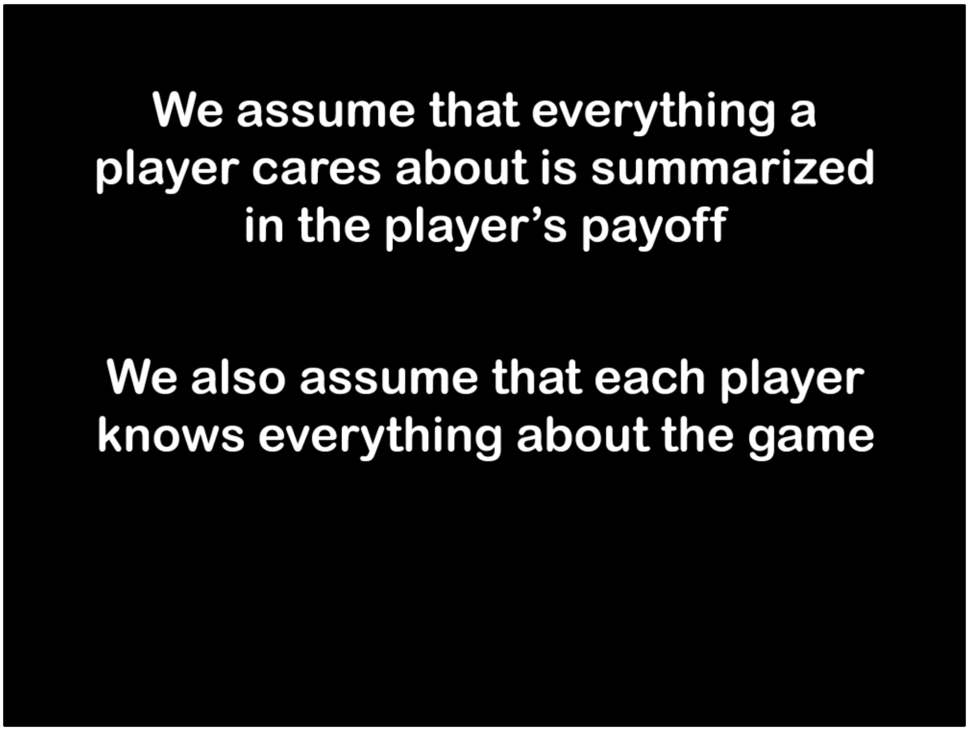
Rewards are happiness.

Using wireless gives happiness 10.

Using fast wired gives happiness 15.

Using slow wired gives happpiness 5

If someone else is using the wired, then the internet is slow for everyone else..



Each player is trying to maximize his or her utility.

This does not necessarily mean the players are selfish. This can be reflected in their utility function.

Prisoner's Dilemma

		Suspect 2: Conf	Suspect 2: No Conf
Suspect 1	Conf	-4,-4	0,-10
	No Conf	-10,0	-1,-1

Utility is -(number of years in prison)

No matter what the other player does, the best thing to do is confess.

Best Responses

A strategy s_1^* is a **best response** by player 1 to a strategy s_2 for player 2 if

$$\pi_1(s_1^*, s_2) \geq \pi_1(s_1, s_2)$$

for all strategies $s_1 \in S_1$.

		Suspect 2	
		Conf	No Conf
Suspect 1	Conf	-4,-4	0,-10
	No Conf	-10,0	-1,-1

If Suspect 2 does not confess, then confessing is a best response for Suspect 1

Dominant Strategy

A strategy s_1^* is a **Dominant Strategy** for player 1 if s_1^* is a Best Response to every possible strategy for player 2.

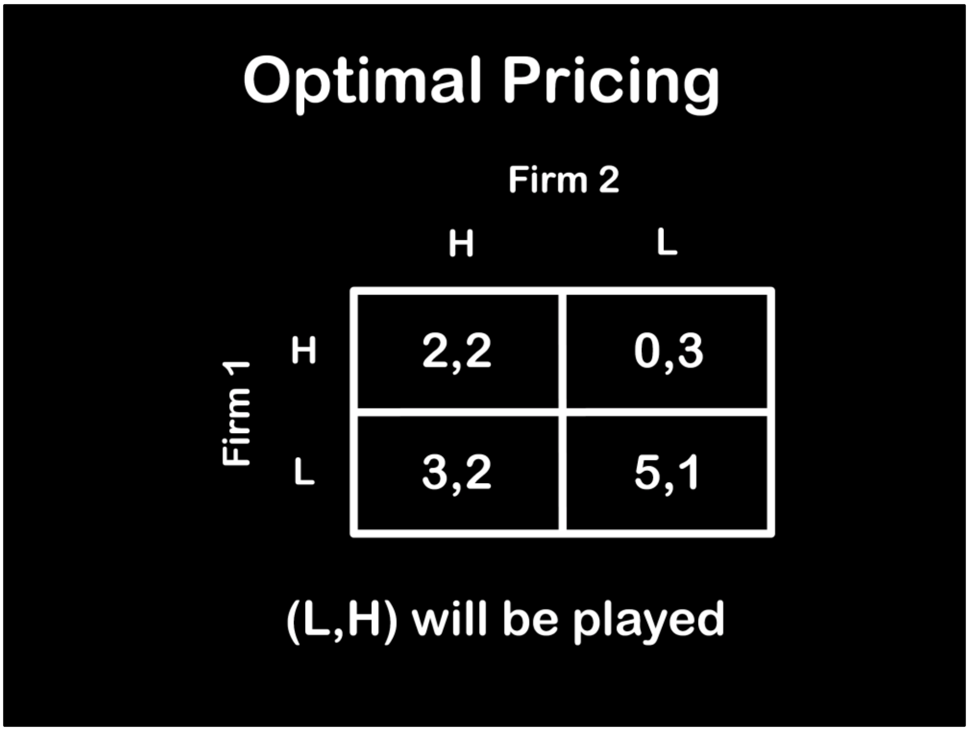
		Suspect 2	
		Conf	No Conf
Suspect 1	Conf	-4,-4	0,-10
	No Conf	-10,0	-1,-1

Confessing is a dominant strategy for both Suspects!

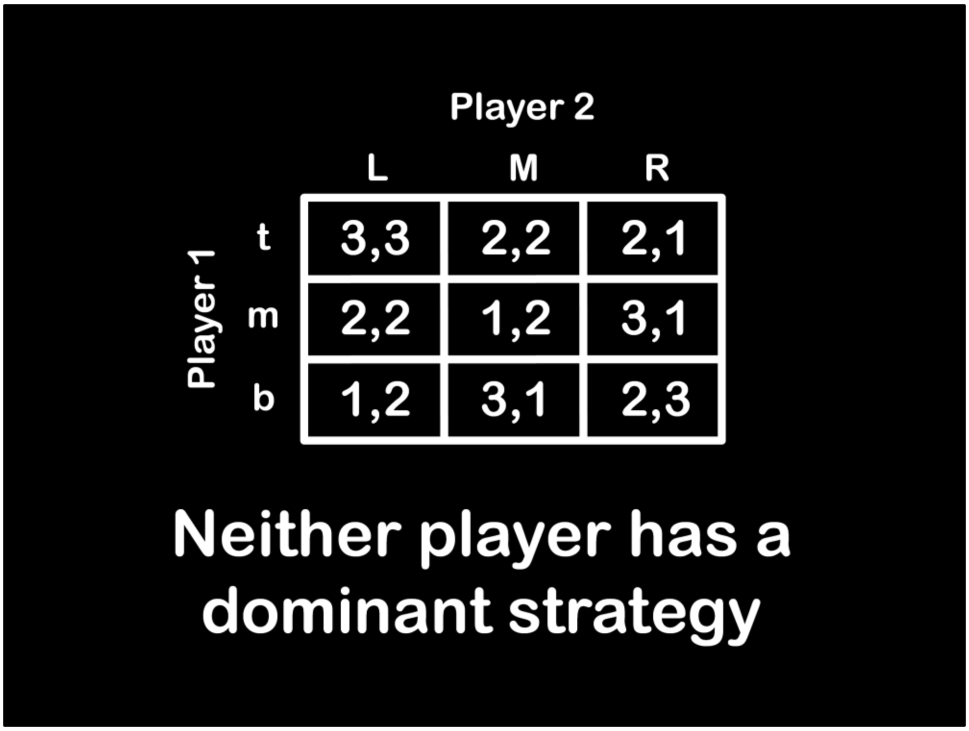
Player 2

Player 1	I	II
I	3,3	1,1
II	1,1	0,0

I is a dominant strategy for both players



There is no dominate strategy for player 2. Player 1 will always play low, and player 2 will play high, since that is the better option when player 1 plays low.



In the majority of game, players don't have dominant strategies.

Nash Equilibrium

A pair of strategies (s_1*,s_2*) is in Nash Equilibrium if s_1* is a Best Response by player 1 to s_2*, and s_2* is a Best Response by player 2 to s_1*.

Player 1 \ Player 2	L	M	R
t	3,3	2,2	2,1
m	2,2	1,2	3,1
b	1,2	3,1	2,3

This means that if, using those strategies, than neither player will gain anything from changing their strategy.

Top left is a Nash equalibirum, as, if they choose (t,L), then neither player will want to change their action. It is a stable situation.

Coordination Game

Player 2

Player 1		L	R
	L	1,1	0,0
	R	0,0	1,1

Nash Equilibria: (L,L), (R,R)

Dove-Hawk

Animal 2

Animal 1		D	H
	D	3,3	1,5
	H	5,1	0,0

Nash Equilibria: (D,H), (H,D)

Matching Pennies

Player 2

Player 1	H	T
H	-1,+1	+1,-1
T	+1,-1	-1,+1

No pure Nash Equilibria Exist!

Randomized Strategies

Player 2

Player 1	H	T
H	-1,+1	+1,-1
T	+1,-1	-1,+1

Player 1 picks H with probability p and
Player 2 picks H with probability q

Player 1 \ Player 2	H	T
H	-1,+1	+1,-1
T	+1,-1	-1,+1

(p=1/2,q=1/2) is an equilibrium!

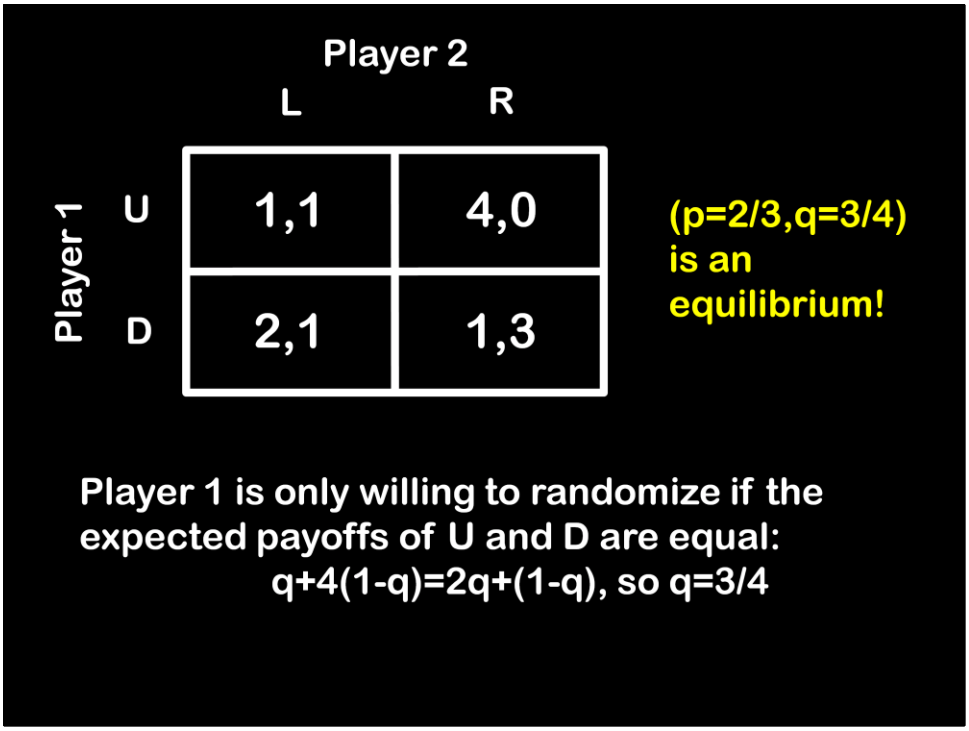
E[Payoff for P1 doing H] = (-1)q + (+1)(1-q) = 1-2q

E[Payoff for P1 doing T] = (+1)q + (-1)(1-q) = 2q-1

Player 1 will choose H if 1-2q > 2q-1. i.e., if q < 1/2

Player 1 will choose T if 1-2q < 2q-1. i.e., if q > 1/2

We say that (p*,q*) is a mixed strategy Nash Equilibrium if p* is a best response by player 1 to q* and q* is a best response by player 2 to p*



E[P1 choosing U] = 4-3q

E[P1 choosing D] = 1 + q

If E[p1 choosing U] = E[p1 choosing D] then q=3/4

E[P2 choosing L] = 1

E[P2 choosing R] = 3-3p

If E[p2 choosing L] = E{P2 choosing R] then p = 2/3

Every game has a randomized nash equalibirum

Kicker \ Goalie	*l*	*r*
L	.58,.42	.95,.05
R	.93,.07	.70,.30

Soccer penalty kick

Player 1 \ Player 2	L	R
U	1,1	3,0
D	0,3	2,2

The only Nash Equilibrium is (U,L)

But (U,L) gives each player a payoff of 1, whereas (D,R) gives them 2.

Nash Equilibrium not always socially optimal g2g

ttyl