Constructive Logic (15-317), Fall 2014
Assignment 4: Classical Logic

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Out: Thursday, September 25, 2014
Due: October 2, 2014 (before class)

In this assignment, you will practice doing constructive logic proofs and further explore the relationship between constructive and classical logic.

The written portion of your work (Section 1) should be submitted electronically before the beginning of class. Please convert your homework to a PDF file titled `hw04.pdf`, and put the file in:

```
/afs/andrew/course/15/317/submit/<your andrew id>
```

If you are familiar with LaTeX, you are encouraged to use this document as a template for typesetting your solutions, but you may alternatively write your solutions *neatly* by hand and scan them.

The Tutch portion of your work (Section 2) should be submitted electronically using the command

```
$ /afs/andrew/course/15/317/bin/submit -r hw04 <files...>
```

from any Andrew server. You may check the status of your submission by running the command

```
$ /afs/andrew/course/15/317/bin/status hw04
```

If you have trouble running either of these commands, email Joe or Evan.

1 Warm-up (3pts)

Tutch allows you to do classical proofs by employing the following rule:

$$\dfrac{\begin{array}{c}[\neg A\ true]^u \\ \vdots \\ \bot\ true\end{array}}{A\ true}\ PBC^u_{Tutch}$$

Task 1 (3 points)**.** Prove that this rule is admissable. That is, show that if $\Gamma, \neg A$ *true* $\vdash_C \bot$ *true*, then $\Gamma \vdash_C A$ *true*, where $\vdash_C$ means "derivable classically". See the end of the assignment for the rules and proof terms of $\vdash_C$.

2 Tutch (16pts)

Classical proofs in Tutch must be declared with `classical proof`. Here's an example, proving double-negation elimination:

```
classical proof dne : ~~A => A =
begin
  [ ~~A;
    % prove A by contradiction:
    [ ~A;
      F ];
    A ];
  ~~A => A
end;
```

Remember, the only additional rule for classical proofs in Tutch is the one mentioned in the previous section. Tutch does not use the *A false* judgment and associated rules we discussed in class.

Task 2 (16 pts)**.** Give *classical* proofs of the following theorems in Tutch.

```
proof negContra : (~A => ~B) => (B => A)
proof selfAbsurd : (~A => A) => A
proof deMorganNotAnd : ~(A & B) => ~A | ~B
proof deMorganNotAll : ~(!x:t. A(x)) => ?x:t. ~A(x)
```

A Classical rules

$$\frac{\Gamma, A\ \mathit{true} \vdash_C B\ \mathit{true}}{\Gamma \vdash_C A \supset B\ \mathit{true}} \supset T \qquad \frac{\Gamma \vdash_C A\ \mathit{true} \quad \Gamma \vdash_C B\ \mathit{true}}{\Gamma \vdash_C A \wedge B\ \mathit{true}} \wedge T \qquad \frac{}{\Gamma \vdash_C \top\ \mathit{true}} \top T$$

$$\frac{\Gamma \vdash_C A\ \mathit{true}}{\Gamma \vdash_C A \vee B\ \mathit{true}} \vee T1 \qquad \frac{\Gamma \vdash_C B\ \mathit{true}}{\Gamma \vdash_C A \vee B\ \mathit{true}} \vee T2 \qquad \frac{\Gamma \vdash_C A\ \mathit{false}}{\Gamma \vdash_C \neg A\ \mathit{true}} \neg T$$

$$\frac{\Gamma \vdash_C A\ \mathit{true} \quad \Gamma \vdash_C B\ \mathit{false}}{\Gamma \vdash_C A \supset B\ \mathit{false}} \supset F \qquad \frac{\Gamma \vdash_C A\ \mathit{false} \quad \Gamma \vdash_C B\ \mathit{false}}{\Gamma \vdash_C A \vee B\ \mathit{false}} \vee F \qquad \frac{}{\Gamma \vdash_C \bot\ \mathit{false}} \bot F$$

$$\frac{\Gamma \vdash_C A\ \mathit{false}}{\Gamma \vdash_C A \wedge B\ \mathit{false}} \wedge F1 \qquad \frac{\Gamma \vdash_C B\ \mathit{false}}{\Gamma \vdash_C A \wedge B\ \mathit{false}} \wedge F2 \qquad \frac{\Gamma \vdash_C A\ \mathit{true}}{\Gamma \vdash_C \neg A\ \mathit{false}} \neg F$$

$$\frac{\Gamma \vdash_C A\ \mathit{true} \quad \Gamma \vdash_C A\ \mathit{false}}{\Gamma \vdash_C \#} \# \qquad \frac{}{\Gamma, A\ \mathit{true} \vdash_C A\ \mathit{true}} \mathsf{hyp}T \qquad \frac{}{\Gamma, A\ \mathit{false} \vdash_C A\ \mathit{false}} \mathsf{hyp}F$$

$$\frac{\Gamma, A\ \mathit{false} \vdash_C \#}{\Gamma \vdash_C A\ \mathit{true}} \mathsf{PBC}T \qquad \frac{\Gamma, A\ \mathit{true} \vdash_C \#}{\Gamma \vdash_C A\ \mathit{false}} \mathsf{PBC}F$$