Constructive Logic (15-317), Fall 2014
Assignment 1: Natural Deduction

Joe Tassarotti (`jtassaro@andrew`), Evan Cavallo (`ecavallo@andrew`)

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Welcome to 15-317, Fall 2014 edition! In this, your first homework assignment, you will practice essential logic skills like carrying out proofs in natural deduction and verifying the local soundness and completeness of logical connectives.

The Tutch portion of your work (Section 1) should be submitted electronically using the command

```
$ /afs/andrew/course/15/317/bin/submit -r hw01 <files...>
```

from any Andrew server. You may check the status of your submission by running the command

```
$ /afs/andrew/course/15/317/bin/status hw01
```

If you have trouble running either of these commands, email Evan or Joe.

The written portion of your work (Sections 2 and 3) should be submitted at the beginning of class. If you are familiar with LaTeX, you are encouraged to use this document as a template for typesetting your solutions, but you may alternatively write your solutions *neatly* by hand.

1 Tutch Proofs (15 points)

Task 1 (15 points)**.** Prove the following theorems using Tutch:

```
orOut : A | B => (A => B) => B
uncurry : (A => B => C) => (A & B => C)
fromNeg : (~A => A) => ~~A
flip : (A => ~B) => (B => ~A)
demorgan : ~A | ~B => ~(A & B)
```

On Andrew machines, you can check your progress against the requirements file `/afs/andrew/course/15/317/req/hw01.req` by running the command

```
$ /afs/andrew/course/15/317/bin/tutch -r hw01 <files...>
```

2 Can't Tutch This (8 points)

Task 2 (4 points). Give a derivation of the following judgement in terms of the inference rule notation given in lecture:

$$((A \wedge B) \vee C) \supset ((A \vee C) \wedge (B \vee C)) \; \mathit{true}$$

Task 3 (4 points). Give a derivation of the following judgement in terms of the inference rule notation given in lecture:

$$(A \vee \neg A) \supset ((A \supset B) \supset B) \supset (A \vee B) \; \mathit{true}$$

3 Harmony (16 points)

Consider two new connectives, $\heartsuit(A, B, C)$ and $\diamond(A, B, C)$, defined by the following introduction and elimination rules.

$$\frac{\begin{array}{cc} [A\ \mathit{true}] & [B\ \mathit{true}] \\ \vdots & \vdots \\ C\ \mathit{true} & C\ \mathit{true} \end{array}}{\heartsuit(A,B,C)\ \mathit{true}}\ \heartsuit I \qquad \frac{\heartsuit(A,B,C)\ \mathit{true} \quad A\ \mathit{true}}{C\ \mathit{true}}\ \heartsuit E_1 \qquad \frac{\heartsuit(A,B,C)\ \mathit{true} \quad B\ \mathit{true}}{C\ \mathit{true}}\ \heartsuit E_2$$

$$\frac{\begin{array}{c} [A\ \mathit{true}] \\ \vdots \\ C\ \mathit{true} \end{array}}{\diamond(A,B,C)\ \mathit{true}}\ \diamond I_1 \qquad \frac{\begin{array}{c} [B\ \mathit{true}] \\ \vdots \\ C\ \mathit{true} \end{array}}{\diamond(A,B,C)\ \mathit{true}}\ \diamond I_2 \qquad \frac{\diamond(A,B,C)\ \mathit{true} \quad A\ \mathit{true} \quad B\ \mathit{true}}{C\ \mathit{true}}\ \diamond E$$

Task 4 (4 points). Are the rules for $\heartsuit$ locally sound? If so, give all local reductions for the rules; if not, explain (informally) why.

Task 5 (4 points). Are the rules for $\heartsuit$ locally complete? If so, give all local reductions for the rules; if not, explain (informally) why.

Task 6 (4 points). Are the rules for $\diamond$ locally sound? If so, give all local reductions for the rules; if not, explain (informally) why.

Task 7 (4 points). Are the rules for $\diamond$ locally complete? If so, give all local reductions for the rules; if not, explain (informally) why.