

1. Indicator Random Variables

100 homeworks are on the table, with two questions to be graded. Andy is in charge of grading question one and Patrick is in charge of grading question two. First, Andy grades some homeworks at random; each homework has probability 0.3 of being graded. Next, Patrick randomly grades half of the homeworks. That is, he grades 50 homeworks. Assume Andy and Patrick make their choices independently.

- (a) Let N be the number of homeworks that Andy graded. What is $E[N]$?

Solution:

- (b) Let M be the number of homeworks that Andy graded and Patrick did not grade. What is $E[M]$?

Solution:

- (c) Let a “good pair” be a pair of adjacent homeworks such that both questions are graded. Let P be the number of good pairs. What is $E[P]$?

Solution:

2. Conditional Probability

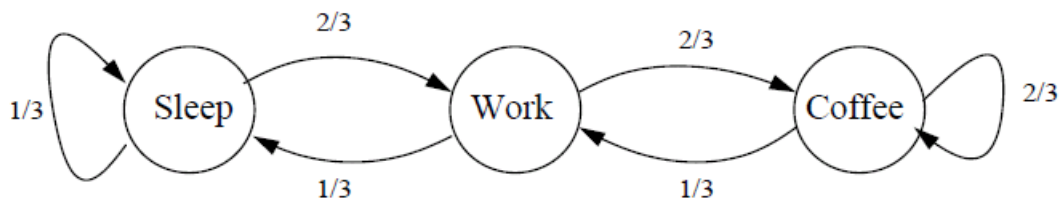
- (a) $Pr[A] = 1/2, Pr[B] = 1/3, Pr[A|B] = 1/4. Pr[A \cap B] = ?$

Solution:

- (b) $Pr[A|B] = 1/2, Pr[B|A] = 1/5, A, B$ are independent. $Pr[A \cap B] = ?$

Solution:

- (c) Peter only sleeps, works, or drinks coffee in his life. Each hour, Peter changes his status according to the following graph. Peter is now at work.



Let T represent the number of hours until he goes to sleep. What is $E[T]$?

Solution: