

HW1 (i) When an ideal gas undergoes an isothermal process, $\Delta U = 0$ and $\Delta H = 0$.

$$w = -nRT \ln \frac{P_1}{P_2} = - (1.64 \text{ mol}) (8.314 \text{ J K}^{-1} \text{ mol}^{-1}) (300 \text{ K}) \ln \frac{15.0 \text{ atm}}{1.00 \text{ atm}} = -6.75 \times 10^3 \text{ J}$$

$$q = \Delta U - w = -w = 6.75 \times 10^3 \text{ J}$$

(ii) $q = 0$ for an adiabatic process.

To determine ΔU and ΔH , T_2 needs to be calculated.

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} = (300 \text{ K}) \left(\frac{1.00 \text{ atm}}{15.0 \text{ atm}} \right)^{(5/3-1)/5/3} = 101.6 \text{ K}$$

Now the rest of the quantities can be calculated:

$$\Delta U = C_V \Delta T = \frac{3}{2} (1.64 \text{ mol}) (8.314 \text{ J K}^{-1} \text{ mol}^{-1}) (101.6 \text{ K} - 300 \text{ K}) = -2.47 \times 10^3 \text{ J}$$

$$w = \Delta U - q = \Delta U = -2.47 \times 10^3 \text{ J}$$

$$\Delta H = C_P \Delta T = \frac{5}{2} (1.64 \text{ mol}) (8.314 \text{ J K}^{-1} \text{ mol}^{-1}) (101.6 \text{ K} - 300 \text{ K}) = -4.12 \times 10^3 \text{ J}$$

(iii) When an ideal gas undergoes an isothermal process, $\Delta U = 0$ and $\Delta H = 0$.

$$w = -P_{\text{ex}} (V_2 - V_1)$$

V_1 and V_2 can be determined using the ideal gas law:

$$V_1 = \frac{nRT}{P_1} = \frac{(1.64 \text{ mol}) (0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1}) (300 \text{ K})}{15.0 \text{ atm}} = 1.641 \text{ L}$$

$$V_2 = \frac{nRT}{P_2} = \frac{(1.64 \text{ mol}) (0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1}) (300 \text{ K})}{1.00 \text{ atm}} = 24.62 \text{ L}$$

Therefore,

$$w = - (1.00 \text{ atm}) (24.62 \text{ L} - 1.641 \text{ L}) \left(\frac{101.3 \text{ J}}{1 \text{ L atm}} \right) = -2.33 \times 10^3 \text{ J}$$

and

$$q = \Delta U - w = -w = 2.33 \times 10^3 \text{ J}$$

For (iv) below, you need to start with $C_V \Delta T = -P_{\text{ext}} \Delta V$ since they are both ΔU . For the gas, $C_V = (3/2)nR$ and

$$\frac{3}{2} nR(T_2 - T_1) = -P_{\text{ext}} \left(\frac{nRT_2}{P_2} - \frac{nRT_1}{P_1} \right) \text{ and } P_2 = P_{\text{ext}}$$

$$\rightarrow \frac{3}{2}(T_2 - T_1) = -(T_2 - P_{ext} \frac{T_1}{P_1})$$

$$T_2 + \frac{3}{2}T_2 = P_{ext} \frac{T_1}{P_1} + \frac{3}{2}T_1$$

$$T_2 = \frac{2}{5} \left(\frac{P_{ext}}{P_1} + \frac{3}{2} \right) T_1$$

(iv) $q = 0$ for an adiabatic process.

To determine ΔU and ΔH , T_2 needs to be calculated. Using the same procedure as

(ii) above

$$\begin{aligned} T_2 &= \frac{2}{5} \left(\frac{P_{ex}}{P_1} + \frac{3}{2} \right) T_1 \\ &= \frac{2}{5} \left(\frac{1.00 \text{ atm}}{15.0 \text{ atm}} + \frac{3}{2} \right) (300 \text{ K}) = 188 \text{ K} \end{aligned}$$

Now the rest of the quantities can be calculated:

$$\Delta U = C_V \Delta T = \frac{3}{2} \text{ (1.64) } (\cancel{1} \text{ mol}) (8.314 \text{ J K}^{-1} \text{ mol}^{-1}) (188 \text{ K} - 300 \text{ K}) = -\cancel{1.40}^{2.30} \times 10^3 \text{ J}$$

$$w = \Delta U - q = \Delta U = -\cancel{1.40}^{2.30} \times 10^3 \text{ J}$$

$$\Delta H = C_P \Delta T = \frac{5}{2} \text{ (1.64) } (\cancel{1} \text{ mol}) (8.314 \text{ J K}^{-1} \text{ mol}^{-1}) (188 \text{ K} - 300 \text{ K}) = -\cancel{2.33}^{3.82} \times 10^3 \text{ J}$$