# HOMEWORK - I - SOLUTIONS

## QUESTION 1:

$Q_n \; \{\phi_{-n}, \ldots, \phi_n\}$ for $-\pi \leq x \leq \pi$

$\phi_0(x) = 1$

$\phi_k(x) = \cos kx$ for $k = 1, \ldots n$

$\phi_{-k}(x) = \sin(-kx)$ for $k = -1, \ldots, -n$

(i) Elements of $F_3$ should satisfy properties of a vector space

let $x$ and $y$ be elements in $F_3$

$$\left.\begin{aligned} x &= a_0\phi_{-3} + a_1\phi_{-2} + a_2\phi_{-1} + \ldots + a_6\phi_3 \\ y &= b_0\phi_{-3} + b_1\phi_{-2} + \ldots + b_6\phi_3 \end{aligned}\right\} \quad a_i, b_i \in \mathbb{R}$$

i- $x+y \in F_3$

$$x+y = (a_0\phi_{-3} + a_1\phi_{-2} + \ldots + a_6\phi_3) + (b_0\phi_{-3} + b_1\phi_{-2} + \ldots + b_6\phi_3)$$

$$= (a_0+b_0)\phi_{-3} + (a_1+b_1)\phi_{-2} + \ldots$$

$F_3 = \sum c_i\phi_i \qquad x+y \in F_3$

ii - $(x+y)+z = x+(y+z)$

$$(x+y)+z = [a_0\phi_{-3} + \ldots + a_6\phi_3] + [b_0\phi_{-3} + \ldots + b_6\phi_3] +$$

$$[e_0\phi_{-3} + \ldots + e_6\phi_3] = \Big([a_0+b_0+e_0]\phi_{-3} + [a_1+b_1+e_1]\phi_{-2} + \ldots\Big) \in F_3$$

iii - $x + y = y + x$

$$\left( a_0 \phi_{-3} + \dots + a_6 \phi_3 \right) + \left( b_0 \phi_{-3} + \dots + b_6 \phi_3 \right) = \left( b_0 \phi_{-3} + \dots + b_6 \phi_3 \right) + \left( a_0 \phi_{-3} + \dots + a_6 \phi_3 \right)$$

$$\in F_3$$

iv - $\exists$ an element $0 \in F_3$ such that $x + 0 = x$

$$\left( a_0 \phi_{-3} + \dots + a_6 \phi_3 \right) + 0 = \left( a_0 \phi_{-3} + \dots + a_6 \phi_3 \right) = x \in F_3$$

v - $\exists$ an element $-x$ such that $x + (-x) = 0$

$$\left( a_0 \phi_{-3} + \dots + a_6 \phi_3 \right) + \left( -a_0 \phi_{-3} - a_6 \phi_3 \right) = 0 \qquad x - x = 0$$

vi - $\alpha x \in F_3$

$$\alpha x = \alpha \left( a_0 \phi_{-3} + \dots + a_6 \phi_3 \right) = \alpha a_0 \phi_{-3} + \dots + \alpha a_6 \phi_3 \in F_3$$

vii - $(x + y)\alpha = \alpha x + \alpha y$

$$\alpha (x + y) = \alpha \left[ \left( a_0 \phi_{-3} + a_1 \phi_{-2} + \dots + a_6 \phi_3 \right) + \left( b_0 \phi_{-3} + \dots b_6 \phi_3 \right) \right]$$

$$= \left( \alpha a_0 \phi_{-3} + \alpha a_1 \phi_{-2} + \dots + \alpha a_6 \phi_3 \right) + \left( \alpha b_0 \phi_{-3} + \dots \right)$$

$$\alpha (x + y) = \alpha x + \alpha y$$

viii - $(\alpha + \beta) x = \alpha x + \beta x$

$$\left( \alpha a_0 \phi_{-3} + \dots + \alpha a_6 \phi_3 + \beta a_0 \phi_{-3} + \dots + \beta a_6 \phi_3 \right) = \alpha x + \beta x$$

$$\alpha \left( a_0 \phi_{-3} + \dots + a_6 \phi_3 \right) + \beta \left( a_0 \phi_{-3} + \dots a_6 \phi_3 \right) \in F_3 .$$

ix - $\alpha(\beta x) = (\alpha\beta)x$

$$\alpha[\beta a_0 \phi_{-3} + \beta a_6 \phi_3] = (\alpha\beta)[a_0\phi_{-3} + a_6\phi_3]$$

$$\alpha\beta a_0 \phi_{-3} + \alpha\beta a_6 \phi_3 = \alpha\beta a_0 \phi_{-3} + \alpha\beta a_6 \phi_3$$ ✓

x - $\alpha x = x$ for $\alpha = 1$

$$\alpha x = \alpha(a_0\phi_{-3} + a_1\phi_{-2} + \dots + a_6\phi_3) = 1(a_0\phi_{-3} + \dots a_6\phi_3)$$

$$= a_0\phi_{-3} + \dots + a_6\phi_3$$

$\alpha x = x$ for $\alpha = 1 \in F_3$

After checking and satisfying these 10 requirements

$\boxed{F_3 \text{ is a vector space.}}$

ii Not all continous fncts for $-\pi \leq x \leq \pi$ lie within $F_3$.

give an example :

$$f(x) = x^2$$

$$f(x) = \sin 5x$$

$$f(x) = x^2 + 4$$

$\vdots$

(iii) Show $(f,g) = \int_{-\pi}^{\pi} f(x)g(x)\,dx$ is an inner prdct:

Let $f(x) = \sum_{i=1}^{3} a_i \phi_i$

$g(x) = \sum_{i=1}^{3} b_i \phi_i$

1) $(f,g)$ should be a real no:

$$\int_{-\pi}^{\pi} f(x)g(x)\,dx = \int_{-\pi}^{\pi} \left(\sum a_i\phi_i\right)\left(\sum b_j\phi_j\right)dx = \int_{-\pi}^{\pi} \sum_i \sum_j \underbrace{a_i b_j \phi_i \phi_j}_{\text{always a real no}} dx$$

$\phi_i(x) =$ cosx or $-$sinx or 1.

2) $(f,f) \geq 0$ and $(f,f) = 0$ iff $f = 0$

$$\int_{-\pi}^{\pi} f(x)f(x)\,dx = \int_{-\pi}^{\pi} \left(\sum_{i=1}^{3} a_i\phi_i\right)^2 dx \Rightarrow \text{always positive due to } (\ )^2.$$

The only way to get 0 from this integral is to set $f=0$ by making the area under integral $= 0$.

3) $(f+g, h) = (f,h) + (g,h)$

$$(f+g,h) = \int_{-\pi}^{\pi} (f(x)+g(x))\,h(x)\,dx$$

$$= \int_{-\pi}^{\pi} [f(x)h(x) + g(x)h(x)]\,dx = \int_{-\pi}^{\pi} f(x)h(x)\,dx + \int_{-\pi}^{\pi} g(x)h(x)\,dx$$

let $h(x) = \sum_{i=1}^{3} c_i\phi_i \quad dx$

$$\int_{-\pi}^{\pi} \sum\sum a_i\phi_i b_i\phi_i + \int_{-\pi}^{\pi} \sum\sum b_i\phi_i c_i\phi_i = \int_{-\pi}^{\pi}\left(\sum\sum a_i b_i \phi_i^2 + \sum\sum b_i c_i \phi_i^2\right)$$ 5

4) $(\lambda f, g) = \lambda (f, g)$

$$\int_{-\pi}^{\pi} \lambda f(x) g(x)\,dx = \lambda\left(\sum a_i\phi_i\right)\sum b_j\phi_j\,dx = \lambda(f,g)$$

5) $(f,g) = g,f)$

$$\int_{-\pi}^{\pi} \sum b_j\phi_j \sum a_i\phi_i\,dx = \int_{-\pi} g(x) f(x)\,dx$$

Satisfying 5 rules defined, $(f,g)$ is an inner prdct in $F_3$.

(iv) Is $(f,g) = \int_{-\pi}^{\pi} e^{-x^2} f(x) g(x)\,dx$ inner product for $F_3$?

$$(f,g) = \int_{-\pi}^{\pi} e^{-x^2}\left[\sum a_i\phi_i\right]\left[\sum b_j\phi_j\right]dx = \sum_i\sum_j \int_{-\pi}^{\pi} e^{-x^2}\phi_i\phi_j\,dx$$

using same f and g from previous part.

is always a real #.

$(f,f) \geq 0$ and $(f,f) = 0$ iff $f = 0$

$(f+g, h) = (f,h) + (g,h)$

$(f,g) = (g,f)$

$(\lambda f, g) = \lambda(f,g)$

$\int_{-\pi}^{\pi} \exp(-x^2).dx = 1.77244$

can be proved similarly such as in part iii)

So $\int_{-\pi}^{\pi} e^{-x^2} f(x) g(x)\,dx$ is an inner product

(v) Finding an orthonormal basis for $F_2$:

$$F_2 \rightarrow \{\phi_{-2}, \phi_{-1}, \phi_0, \phi_1, \phi_2\}$$

$$\phi_{-2}(x) = -\sin 2x$$
$$\phi_{-1}(x) = -\sin x$$
$$\phi_0(x) = 1$$
$$\phi_1(x) = \cos x$$
$$\phi_2(x) = \cos 2x$$

1) Linear independence:

$$\{c_1\phi_1 + c_2\phi_2 + c_0\phi_0 + c_{-1}\phi_{-1} + c_{-2}\phi_{-2}\} = F_2$$

2) Orthonormality:

$$\vec{x_i} \cdot \vec{x_j} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{1}{4}\sin 2x$$

$$\int \cos^2 x \, dx = \frac{1}{2}(x + \cos x . \sin x)$$

$$\int_{-\pi}^{\pi} f(x)\, g(x)\, dx = (f \cdot g)$$

starting with ~~1 and~~ $\sin x$ $\rightarrow$ $\int_{-\pi}^{\pi} c.\sin x . c.\sin x \, dx = 1$

$$c^2 \int_{-\pi}^{\pi} \sin x . \sin x \, dx = 1 \qquad c^2 \int_{-\pi}^{\pi} \sin^2 x \, dx = 1$$

$$c^2 . \pi = 1 \qquad c = 1/\sqrt{\pi}$$

$$\downarrow$$

$$\boxed{\frac{1}{\sqrt{\pi}} \sin x}$$

$\cos x$:

$$(c')^2 \int_{-\pi}^{\pi} \cos^2 x \, dx = 1 \qquad (c')^2 \cdot \pi = 1 \qquad c' = 1/\sqrt{\pi} \rightarrow \boxed{\frac{1}{\sqrt{\pi}} \cos x}$$

$\sin 2x$:

$$(c'')^2 \int_{-\pi}^{\pi} \sin^2 2x \, dx = 1 \qquad c''^2 = 1/\pi \qquad c'' = \sqrt{1/\pi}$$

$$\rightarrow \frac{1}{\sqrt{\pi}} \sin 2x$$

$\cos 2x$:

$$(c''')^2 \int_{-\pi}^{\pi} \cos^2 2x \, dx = 1 \qquad (c''')^2 = 1/\pi \qquad (c''') = 1/\sqrt{\pi} \rightarrow \frac{1}{\sqrt{\pi}} \cos 2x$$

3) Orthogonality must be checked by cross terms:

$$\int_{-\pi}^{\pi} \frac{1}{\sqrt{\pi}} \sin x \cdot \frac{1}{\sqrt{\pi}} \cos x = 0$$

$$\int_{-\pi}^{\pi} \frac{1}{\sqrt{\pi}} \sin 2x \cdot \frac{1}{\sqrt{\pi}} \sin x \, dx = \int_{-\pi}^{\pi} \frac{1}{\sqrt{\pi}} \sin 2x \cdot \cos x \, dx = 0$$

$$\int_{-\pi}^{\pi} \frac{1}{\sqrt{\pi}} \cos 2x \cdot \frac{1}{\sqrt{\pi}} \cos x \, dx = 0 = \int_{-\pi}^{\pi} \frac{1}{\sqrt{\pi}} \sin 2x \cdot \cos 2x \, dx = 0$$

So Basis can be written from these facts in pairs:

$$\frac{1}{\sqrt{\pi}} \sin x, \quad \frac{1}{\sqrt{\pi}} \cos x, \quad \frac{1}{\sqrt{\pi}} \sin 2x, \quad \frac{1}{\sqrt{\pi}} \cos 2x \quad \ldots$$

QUESTION 2:

let V be a set of all (3x2) matrices

(i) $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$

i - $A + B = \begin{bmatrix} a_{11}+b_{11} & .. \\ .. & .. \\ .. & a_{32}+b_{32} \end{bmatrix} \in V$

ii - $(A+B)+C = A+(B+C)$

$$\left(\begin{bmatrix} a_{11} & .. \\ .. & .. \\ .. & a_{32} \end{bmatrix} + \begin{bmatrix} b_{11} & .. \\ .. & .. \\ .. & b_{32} \end{bmatrix}\right) + \begin{bmatrix} c_{11} & .. \\ .. & .. \\ . & c_{32} \end{bmatrix} = \begin{bmatrix} a_{11} & .. \\ .. & .. \\ .. & a_{32} \end{bmatrix} + \left(\begin{bmatrix} b_{11}+c_{11} & .. \\ .. & .. \\ .. & b_{32}+c_{32} \end{bmatrix}\right)$$

$\in V$

iii - $(A+B) = (B+A)$

iv - $\exists$ an element $0 \in V$ such that $A+0=A$.

v - $\exists$ an element $(-A)$ such that $A - A = 0$

vi - $\alpha A \in V$

vii - $\alpha(A+B) = \alpha A + \alpha B$

viii - $\alpha(A) + \beta(A) = (\alpha+\beta)A$

ix - $\alpha(\beta A) = \alpha\beta A$

x - $\alpha A = A$ for $\alpha = 1$

can be shown such as Prb 1 part i

$\boxed{\text{So V is a vector space.}}$

give 2 different bases for space :

By definition, any two bases for a $V$ have the same # of vectors. This number is defined as dimension of space. $V_{3\times2}$ has 6 dimension and two bases can be :

$$\rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 10 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 7 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 11 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 12 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 40 \end{bmatrix}.$$

These basis are linearly independent !

QUESTION 3:

(i) Please see Mathematica solution

for $b_1$ $\quad x^1 = \begin{bmatrix} 1.4706 \\ -0.1176 \\ 0.7647 \\ -0.3529 \end{bmatrix}$ or $\begin{bmatrix} 25/17 \\ -2/17 \\ 13/17 \\ -6/17 \end{bmatrix}$

$b_2$ $\quad x^2 = \begin{bmatrix} -4.2353 \\ 1.0588 \\ -2.8824 \\ 1.1765 \end{bmatrix}$ or $\begin{bmatrix} -72/17 \\ 18/17 \\ -49/17 \\ 20/17 \end{bmatrix}$

$b_3$ $\quad x^3 = \begin{bmatrix} 1.8824 \\ -0.4706 \\ 1.0588 \\ -0.4118 \end{bmatrix}$ or $\begin{bmatrix} 32/17 \\ -8/17 \\ 18/17 \\ -7/17 \end{bmatrix}$

$b_4$ $\quad x^4 = \begin{bmatrix} +0.0882 \\ -0.1471 \\ 0.2059 \\ 0.0588 \end{bmatrix}$ or $\begin{bmatrix} +3/34 \\ -5/34 \\ 7/34 \\ 1/17 \end{bmatrix}$

(ii) $A^4 = \begin{bmatrix} 525 & 255 & 932 & 4051 \\ 27 & 556 & 835 & 1623 \\ -368 & 1098 & 1182 & 452 \\ 680 & 442 & 1450 & 5720 \end{bmatrix}$

In[22]:=
```
(* Solution to Problem 3 in Assignment 1 - 06-713 *)
(* From Ax=b we have to Solve for x *)
A = {{1, 5, 2, 4}, {1, 0, -1, 2}, {2, -4, -4, 1}, {0, 2, 4, 8}}
(* part1 *)
b = {{1}, {0}, {0}, {0}}
(* x = (A^-1) * b *)
Inverse[A].b
```

Out[22]= {{1, 5, 2, 4}, {1, 0, -1, 2}, {2, -4, -4, 1}, {0, 2, 4, 8}}

{{1}, {0}, {0}, {0}}

ANS :- $\left\{\left\{\frac{25}{17}\right\}, \left\{-\frac{2}{17}\right\}, \left\{\frac{13}{17}\right\}, \left\{-\frac{6}{17}\right\}\right\}$ ✓

```
c = {{0}, {1}, {0}, {0}}
Inverse[A].c
```

Out[28]= $\left\{\left\{\frac{25}{17}\right\}, \left\{-\frac{2}{17}\right\}, \left\{\frac{13}{17}\right\}, \left\{-\frac{6}{17}\right\}\right\}$

Out[29]= {{0}, {1}, {0}, {0}}

ANS : ~~In[31]:=~~ $\left\{\left\{-\frac{72}{17}\right\}, \left\{\frac{18}{17}\right\}, \left\{-\frac{49}{17}\right\}, \left\{\frac{20}{17}\right\}\right\}$ ✓

```
d = {{0}, {0}, {1}, {0}}
Inverse[A].d
```

Out[31]= $\left\{\left\{-\frac{72}{17}\right\}, \left\{\frac{18}{17}\right\}, \left\{-\frac{49}{17}\right\}, \left\{\frac{20}{17}\right\}\right\}$

Out[32]= {{0}, {0}, {1}, {0}}

NS : ~~In[34]:=~~ $\left\{\left\{\frac{32}{17}\right\}, \left\{-\frac{8}{17}\right\}, \left\{\frac{18}{17}\right\}, \left\{-\frac{7}{17}\right\}\right\}$ ✓

```
e = {{0}, {0}, {0}, {1}}
Inverse[A].e
```

Out[34]= $\left\{\left\{\frac{32}{17}\right\}, \left\{-\frac{8}{17}\right\}, \left\{\frac{18}{17}\right\}, \left\{-\frac{7}{17}\right\}\right\}$ ✓

Out[35]= {{0}, {0}, {0}, {1}}

ANS :- $\left\{\left\{\frac{3}{34}\right\}, \left\{-\frac{5}{34}\right\}, \left\{\frac{7}{34}\right\}, \left\{\frac{1}{17}\right\}\right\}$ ✓

```
In[37]:= (* Solution to Problem 3 Part 2 *)
A = {{1, 5, 2, 4}, {1, 0, -1, 2}, {2, -4, -4, 1}, {0, 2, 4, 8}}
(* A^4 is given by *)
A.A.A.A

Out[37]= {{1, 5, 2, 4}, {1, 0, -1, 2}, {2, -4, -4, 1}, {0, 2, 4, 8}}
```

ANS. :- {{525, 255, 932, 4051}, {27, 556, 835, 1623}, {-368, 1098, 1182, 452}, {680, 442, 1450, 5720}} iii

$$A^T = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 5 & 0 & -4 & 2 \\ 2 & -1 & -4 & 4 \\ 4 & 2 & 1 & 8 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 1 & 10 & 6 \\ 0 & 1 & 3 & 1 \\ -6 & 2 & -1 & -3 \\ 6 & 0 & 10 & 12 \end{bmatrix}$$

$$AB^T = \begin{bmatrix} 8 & 0 & -6 & 6 \\ 1 & 1 & 2 & 0 \\ 10 & 3 & -1 & 10 \\ 6 & 1 & -3 & 12 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 8 & 0 & -6 & 6 \\ 1 & 1 & 2 & 0 \\ 10 & 3 & -1 & 10 \\ 6 & 1 & -3 & 12 \end{bmatrix}$$

$$\boxed{(AB)^T = B^T A^T}$$ ✓

QUESTION 4 :

(i) $px_1 + 2x_2 - x_3 = 0$

$2x_1 - x_2 + 3x_3 = 0$

$9x_1 - 4x_2 + 2x_3 = 0$

$\vec{A}\vec{x} = 0$ has a solution / solutions other than zero iff $r(A) < n$

$r(A)$ = rank of A  $n = 3$  so $r(A) < 3$ must be satisfied.

$$\begin{matrix} p & 2 & -1 \\ 2 & -1 & 3 \\ 9 & -4 & 2 \end{matrix} \rightarrow \begin{matrix} 1 & 2/p & -1/p \\ 1 & -1/2 & 3/2 \\ 1 & -4/9 & 2/9 \end{matrix} \rightarrow \begin{matrix} 1 & 2/p & -1/p \\ 0 & -\frac{1}{2}-\frac{2}{p} & \frac{3}{2}+\frac{1}{p} \\ 0 & -\frac{4}{9}-\frac{2}{p} & \frac{2}{9}+\frac{1}{p} \end{matrix}$$

$$\begin{matrix} 1 & 2/p & -1/p \\ 0 & \left(-\frac{1}{2}-\frac{2}{p}\right) & \left(\frac{3}{2}+\frac{1}{p}\right) \\ 0 & \left(-\frac{4}{9}-\frac{2}{p}\right) & \left(\frac{2}{9}+\frac{1}{p}\right) \end{matrix} \rightarrow \begin{matrix} 1 & 2/p & -1/p \\ 0 & 1 & \dfrac{3/2+1/p}{-1/2-2/p} \\ 0 & 1 & \dfrac{2/9+1/p}{-4/9-2/p} \end{matrix}$$

$$\begin{matrix} 1 & 2/p & -1/p \\ 0 & 1 & \dfrac{3/2+1/p}{-1/2-2/p} \\ 0 & 0 & \left(\dfrac{\frac{2}{9}+\frac{1}{p}}{-\frac{4}{9}-\frac{2}{p}}\right)-\left(\dfrac{3/2+1/p}{-1/2-2/p}\right) \end{matrix} \rightarrow$$

if this line is zero then $r < 3$ $(r = 2)$

$$\left(\frac{2}{9}+\frac{1}{p}\right) \Big/ \frac{4}{9}+\frac{2}{p} = \frac{+3/2 + 1/p}{1/2 + 2/p}$$

Solwing $\dfrac{2p+q}{4p+2q} = \dfrac{3p+2}{p+4}$

$= -2p^2 - qp - 8p - 4q + 12p^2 + 8p + 6qp + 4q = 0$

$= 10p^2 + 5pq = 0$

$5p(q+2p) = 0 \longrightarrow \boxed{\begin{array}{l} p=0 \\ q=-2p \text{ or } p=-q/2 \end{array}}$

we have nontrivial sol. in these cases.

ii

$px_1 + x_2 - 3x_2 = 0$

$x_1 - 2x_2 - x_3 = 0$ similarly

$x_1 - qx_2 + x_3 = 0$

$$\begin{array}{ccc} p & 1 & -3 \\ 1 & -2 & -1 \\ 1 & -q & +1 \end{array} \rightarrow \begin{array}{ccc} 1 & 1/p & -3/p \\ 0 & 1 & -2/q-2 \\ 0 & 0 & \dfrac{p+3}{-qp-1} + \dfrac{2}{q-2} \end{array}$$

$\dfrac{p+3}{qp+1} = \dfrac{2}{q-2}$ $\quad qp = -8 + 3q - 2p$

$\boxed{p = \dfrac{3q-8}{2+q}}$ or $q = \dfrac{8+2p}{3-p}$

we have nontrivial sol.

$\left.\begin{array}{l} q=-2 \\ p=3 \end{array}\right] \to \infty$