

## Homework Set #5

(due October 2, 1997)

- 1.) Consider the problem of potential flow around a sphere of radius  $R$ . For axisymmetric flow around a sphere, the stream function is related to velocity by

$$\mathbf{v} = \text{curl} \left\{ \frac{\psi(r, \theta) \mathbf{e}_\phi}{r \sin \theta} \right\}$$

- a. Write down the differential equation and boundary conditions which the stream function must satisfy in this problem. In particular, show that

$$\psi \rightarrow Ur^2 \sin^2 \theta \quad \text{as } r \rightarrow \infty$$

corresponds to  $\mathbf{v} \rightarrow U\mathbf{k}$  and no flow into the sphere requires  $\partial\psi/\partial\theta = 0$  at  $r=R$ .

- b. Deduce a physical meaning for the stream function in this geometry.

**HINTS:** what is the flowrate in a tube formed by rotating a streamline ( $\psi = \text{const}$ ) about the  $z$ -axis? To answer this question, consider the boundary condition for  $\psi$  as  $r \rightarrow \infty$  in which limit the tube becomes a circular cylinder.

**ANSWER:** 
$$\psi = \frac{Q}{2\pi}$$

- c. Solve for the stream function  $\psi(r, \theta)$  and deduce the velocity profile.

**ANSWER:** 
$$\psi(r, \theta) = \frac{1}{2}UR^2 \left[ \left( \frac{r}{R} \right)^2 - \left( \frac{R}{r} \right) \right] \sin^2 \theta$$

- 2.) In the most general statement of Hooke's Law of Elastic Solids, stress and strain are related by:

$$\underline{\mathbf{T}} = 2\eta\underline{\boldsymbol{\epsilon}} + \lambda(\underline{\boldsymbol{\epsilon}} : \underline{\mathbf{D}})\underline{\mathbf{I}} \quad (1)$$

where  $\eta$  and  $\lambda$  are two independent material constants.

- a. Uniaxial stress applied along the  $x$ -axis produces normal strain along all three axes, which are related according to  $\epsilon_{zz} = \epsilon_{yy} = -\nu\epsilon_{xx}$ , where  $\nu$  is a material property called "Poisson's ratio." Expand (1) to obtain expressions for  $T_{xx}$  and  $T_{yy}$  in terms of  $\epsilon_{xx}$  and the material properties.
- b. Recognizing that  $T_{yy} = 0$  for uniaxial stress along the  $x$ -axis, find a relationship between Poisson's ratio and the material constants  $\eta$  and  $\lambda$ .

**ANSWER:** 
$$\nu = \frac{\lambda}{2(\lambda + \eta)}$$

- c. Relate Young's modulus,  $E$ , for pure uniaxial stress to the material constants  $\eta$  and  $\lambda$ .
- d. For the second case of pure shear, expand (1) to obtain expressions for  $T_{xy}$  in terms of  $\epsilon_{xy}$  and the material properties. Relate  $G$ , the modulus of elasticity for pure strain, to  $\eta$  and  $\lambda$ .
- e. Show that  $\nu = \frac{E}{2G} - 1$ .