Summary of Differential Operations Involving the ∇-Operator in Rectangular Coordinates (x, y, z)

$$(\nabla \cdot \boldsymbol{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$(\nabla^2 s) = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\begin{aligned}(\boldsymbol{\tau} : \nabla \boldsymbol{v}) = {} & \tau_{xx}\left(\frac{\partial v_x}{\partial x}\right) + \tau_{xy}\left(\frac{\partial v_x}{\partial y}\right) + \tau_{xz}\left(\frac{\partial v_x}{\partial z}\right) \\ & + \tau_{yx}\left(\frac{\partial v_y}{\partial x}\right) + \tau_{yy}\left(\frac{\partial v_y}{\partial y}\right) + \tau_{yz}\left(\frac{\partial v_y}{\partial z}\right) \\ & + \tau_{zx}\left(\frac{\partial v_z}{\partial x}\right) + \tau_{zy}\left(\frac{\partial v_z}{\partial y}\right) + \tau_{zz}\left(\frac{\partial v_z}{\partial z}\right)\end{aligned}$$

$$[\nabla s]_x = \frac{\partial s}{\partial x} \qquad \text{(D)} \qquad [\nabla \times \boldsymbol{v}]_x = \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}$$

$$[\nabla s]_y = \frac{\partial s}{\partial y} \qquad \text{(E)} \qquad [\nabla \times \boldsymbol{v}]_y = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}$$

$$[\nabla s]_z = \frac{\partial s}{\partial z} \qquad \text{(F)} \qquad [\nabla \times \boldsymbol{v}]_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\nabla^2 \boldsymbol{v}]_x = \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}$$

$$[\nabla^2 \boldsymbol{v}]_y = \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}$$

$$[\nabla^2 \boldsymbol{v}]_z = \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{w}]_x = v_x\left(\frac{\partial w_x}{\partial x}\right) + v_y\left(\frac{\partial w_x}{\partial y}\right) + v_z\left(\frac{\partial w_x}{\partial z}\right)$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{w}]_y = v_x\left(\frac{\partial w_y}{\partial x}\right) + v_y\left(\frac{\partial w_y}{\partial y}\right) + v_z\left(\frac{\partial w_y}{\partial z}\right)$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{w}]_z = v_x\left(\frac{\partial w_z}{\partial x}\right) + v_y\left(\frac{\partial w_z}{\partial y}\right) + v_z\left(\frac{\partial w_z}{\partial z}\right)$$

$$\{\nabla \boldsymbol{v}\}_{xx} = \frac{\partial v_x}{\partial x}$$

$$\{\nabla \boldsymbol{v}\}_{xy} = \frac{\partial v_y}{\partial x}$$

$$\{\nabla \boldsymbol{v}\}_{xz} = \frac{\partial v_z}{\partial x}$$

$$\{\nabla \boldsymbol{v}\}_{yx} = \frac{\partial v_x}{\partial y}$$

$$\{\nabla \boldsymbol{v}\}_{yy} = \frac{\partial v_y}{\partial y}$$

$$\{\nabla \boldsymbol{v}\}_{yz} = \frac{\partial v_z}{\partial y}$$

$$\{\nabla \boldsymbol{v}\}_{zx} = \frac{\partial v_x}{\partial z}$$

$$\{\nabla \boldsymbol{v}\}_{zy} = \frac{\partial v_y}{\partial z}$$

$$\{\nabla \boldsymbol{v}\}_{zz} = \frac{\partial v_z}{\partial z}$$

$$\{\boldsymbol{v} \cdot \nabla \boldsymbol{\tau}\}_{xx} = (\boldsymbol{v} \cdot \nabla)\tau_{xx}$$

$$\{\boldsymbol{v} \cdot \nabla \boldsymbol{\tau}\}_{xy} = (\boldsymbol{v} \cdot \nabla)\tau_{xy}$$

$$\{\boldsymbol{v} \cdot \nabla \boldsymbol{\tau}\}_{xz} = (\boldsymbol{v} \cdot \nabla)\tau_{xz}$$

$$\{\boldsymbol{v} \cdot \nabla \boldsymbol{\tau}\}_{yx} = (\boldsymbol{v} \cdot \nabla)\tau_{yx}$$

$$\{\boldsymbol{v} \cdot \nabla \boldsymbol{\tau}\}_{yy} = (\boldsymbol{v} \cdot \nabla)\tau_{yy}$$

$$\{\boldsymbol{v} \cdot \nabla \boldsymbol{\tau}\}_{yz} = (\boldsymbol{v} \cdot \nabla)\tau_{yz}$$

$$\{\boldsymbol{v} \cdot \nabla \boldsymbol{\tau}\}_{zx} = (\boldsymbol{v} \cdot \nabla)\tau_{zx}$$

$$\{\boldsymbol{v} \cdot \nabla \boldsymbol{\tau}\}_{zy} = (\boldsymbol{v} \cdot \nabla)\tau_{zy}$$

$$\{\boldsymbol{v} \cdot \nabla \boldsymbol{\tau}\}_{zz} = (\boldsymbol{v} \cdot \nabla)\tau_{zz}$$

where the operator $(\boldsymbol{v} \cdot \nabla) = v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$