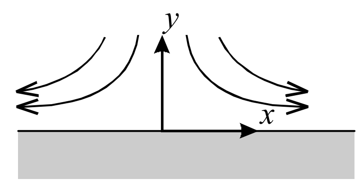
Homework Set #2

(due September 12, 2001)

1.) The velocity profile for 2D stagnation flow is given by:



$$v_x = ax$$
$$v_y = -ay$$

where $a>0$ is a constant with respect to both time and position. Evaluate the following quantities at the point (1,1):

a. $\partial \mathbf{v}/\partial t$ and

b. $D\mathbf{v}/Dt$

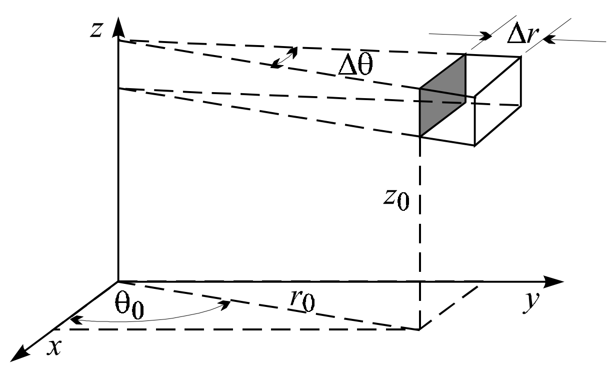
2.) Use the *implicit* definition of gradient (i.e. $df \equiv d\mathbf{r}_p \cdot \nabla f$) to evaluate ∇f in cylindrical coordinates (r, θ, z). **Hint**: referring to p12 of the 2000 Notes, the displacement vector in cylindrical coordinates is

$$d\mathbf{r}_p = dr\,\mathbf{e}_r + r\,d\theta\,\mathbf{e}_\theta + dz\,\mathbf{e}_z$$

Answer:
$$\nabla f = \frac{\partial f}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\mathbf{e}_\theta + \frac{\partial f}{\partial z}\mathbf{e}_z$$

3.) Use the *explicit* definition of gradient

$$\nabla f \equiv \lim_{V \to 0}\left\{\frac{1}{V}\int_A \mathbf{n} f da\right\}$$



to evaluate ∇f in cylindrical coordinates. Your answer should be identical to that in Prob. 2.)

[***Hint***: As an intermediate result, you should obtain the following contribution from the two θ=const. faces:

$$\frac{1}{r}\frac{\partial}{\partial \theta}\left(\mathbf{e}_\theta f\right)$$

Don't forget that $\mathbf{e}_\theta = \mathbf{e}_\theta(\theta)$. The contribution from the two r=const. faces is

$$\frac{1}{r}\frac{\partial (rf)}{\partial r}\mathbf{e}_r(\theta)$$

Don't forget that r is not the same on the two r=const. faces. Also the two r=const. faces are curved.]

4.) If $\mathbf{v} = (z-y)\mathbf{i} + (x-z)\mathbf{j} + (y-z)\mathbf{k}$, compute $\nabla \cdot \mathbf{v}$.

5.) If s (scalar), **u**, and **v** (vectors) are continuously differentiable, verify in Cartesian coordinates that:

a. $\nabla \cdot (\mathbf{u}+\mathbf{v}) = \nabla \cdot \mathbf{u} + \nabla \cdot \mathbf{v}$

b. $\nabla \cdot (s\mathbf{u}) = \nabla s \cdot \mathbf{u} + s(\nabla \cdot \mathbf{u})$

c. $\nabla \cdot (\mathbf{u}\times\mathbf{v}) = \mathbf{v} \cdot (\nabla\times\mathbf{u}) - \mathbf{u} \cdot (\nabla\times\mathbf{v})$

d. $\nabla \cdot (\nabla\times\mathbf{v}) = 0$

e. $\nabla\times(\nabla s) = \mathbf{0}$