Homework Set #10

(due December 10, 2001)

1.) The lubrication analysis for the slider block yields the following expression for the lift force:

$$\frac{F_y}{W} = \frac{6\mu U L^2}{(h_1 - h_2)^2}\left[\ln\frac{h_1}{h_2} - \frac{2(h_1 - h_2)}{h_1 + h_2}\right]$$

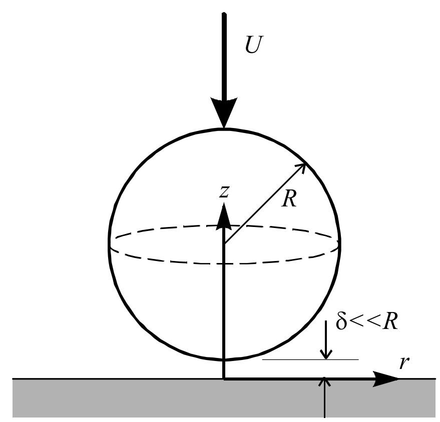
For $h_1 = h_2$, the slider is parallel to the plate and the lift force should become zero, although the expression above is indeterminant. Resolve the indeterminacy by expanding each term in a Taylor series about $\varepsilon=0$, where ε is defined such that $h_1 = h_2(1+\varepsilon)$. Thus obtain the leading term of the asymptotic behavior of the lift force as $\alpha \to 0$.

Hint: It turns out that the difference between the two terms in square brackets is $O(\varepsilon^3)$. Thus you will need to form 3-term Taylor series expansions for both terms. Note that

$$\ln(1+x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \ldots$$

Answer: $$\frac{F_y}{W} = \frac{1}{2}\mu U\left(\frac{L}{h}\right)^3 \alpha \text{ where } \alpha = \frac{h_1 - h_2}{L}$$

2.) Consider the squeezing flow generated by pushing a rigid sphere of radius R though a viscous fluid (viscosity μ) towards a flat plate at speed U. <u>Calculate the force</u> required for this motion when the gap between the sphere and the plate is very small compared to the radius.

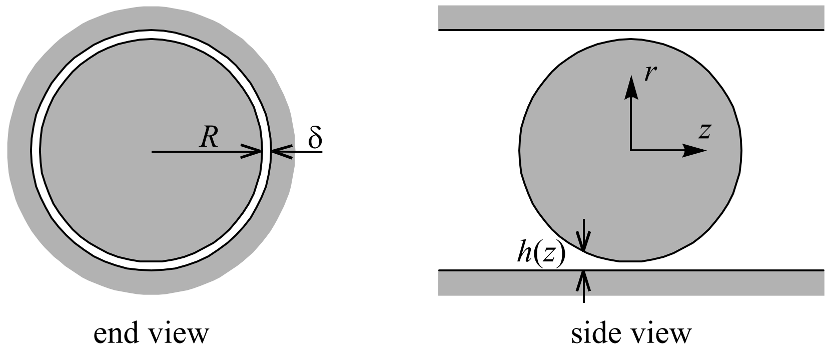


Hint: the pressure profile can be obtained by integrating Reynolds Lubrication Equation. Use cylindrical coordinates (r,z) with the origin located on the plate (as shown at right). Then the pressure profile is a function of only one variable: $p = p(r)$. Following an analysis like that leading to equation (163) of the 2000 Notes, we obtain an equation for the film thickness in the narrowest part of the film (i.e. for $r \ll R$):

$$h(r) = \delta + \frac{r^2}{2R}$$

To obtain the force, you will need to integrate with respect to r. Any integral with respect to r can be more easily evaluated by transforming into an integal with respect to h using $r\,dr = R\,dh$, which is just a rearrangement of $dh/dr = r/R$.

Answer: $$F_z = 6\pi\mu U R \frac{R}{\delta}$$



3.) A sphere of radius R is located inside a slightly larger tube (tube radius is $R+\delta$). Find the force required to move the sphere along the tube at speed U. The fluid occupying the space between the sphere and the tube wall has viscosity μ and density ρ. Both ends of the tube are open to the atmosphere so the pressure of the fluid outside the gap remains near atmospheric on either side of the sphere.

Hint #1. You may assume that $\delta \ll R$ and that the sphere's center remains on the centerline of the tube. The gap between the sphere and the plate can be approximated (in the vicinity of $z=0$) by

$$h(z) = \delta + \frac{z^2}{2R}$$

which also leads to the following approximations:

$$\int_{-\infty}^{\infty} \frac{dz}{h} = \pi\sqrt{\frac{2R}{\delta}} \qquad \int_{-\infty}^{\infty} \frac{dz}{h^2} = \frac{\pi}{2\delta}\sqrt{\frac{2R}{\delta}} \qquad \int_{-\infty}^{\infty} \frac{dz}{h^3} = \frac{3\pi}{8\delta^2}\sqrt{\frac{2R}{\delta}}$$

Hint #2. The velocity and pressure profiles can more easily be approximated by neglecting the curvature of the tube wall; then the profiles in the narrow gap are essentially the same as in 2-D flow between a cylinder of radius R and a flat plate. In particular, the velocity profile for 2-D flow of the cylinder (whose axis is the z-axis) in the x-direction is a linear combination of linear shear flow and parabolic pressure-driven flow:

$$v_x^{LS} = U\frac{y}{h} \quad \text{and} \quad v_x^{PD} = \frac{1}{2\mu}\frac{dp}{dx}y(y-h)$$

Note: the x,y in the above velocity profiles are not the same x,y as in the figures above.

4.) Turbulent flow of water (viscosity = 1 centipoise; density = 1 gram/cm^3) in a 5-cm (diameter) pipe occurs at a cross-sectional averaged velocity of 200 cm/sec.

a. Find the pressure drop using Prandtl's universal law of friction.

b. Estimate the thickness of the laminar sublayer.