Homework Set #1

(due September 5, 2001)

1. Given vectors **A** = **i**+2**j**+3**k** and **B** = -**i**+2**j**+**k**:

   a. Find the angle between **A** and **B**.

   b. Find a third vector which is perpendicular to both **A** and **B**.

2. The plane *ABC* contains the following points (3,0,-1), (3,2,0) and (1,1,-2).

   a. Find any vector perpendicular to this plane. (***Hint***: recall the geometric meaning of cross product.)

   b. Find the projection of **A** on the plane *ABC*, where **A** is the vector drawn from the origin to (3,0,-1). (Note: "projection" refers to the length of a line drawn on the plane connecting two points which are projections normal to the plane of the two ends of the vector.)

3. Find the angle between the normals to the cylinder $x^2 + y^2 = a^2$ and the sphere $(x-a)^2 + y^2 + z^2 = a^2$ at their common point $(a/2, a\sqrt{3}/2, 0)$. (**Hint**: recall that ∇f at some point is normal to a $f(x,y,z)$ = constant surface passing through the same point.)

4. The temperature profile in a certain body is given by $T = xy^2 - z$. At the point (1,0,0), find the unit vector point in the direction in which temperature increases fastest.

5. In parametric form, the trajectory of a particle moving in the *xy* plane is given by $x = t^2$, $y = 2t$. Express its velocity and acceleration in terms of the unit vectors.

6. Given the following:

$$\mathbf{a} = 2\mathbf{i}+\mathbf{j}-2\mathbf{k}$$

$$\mathbf{b} = -2\mathbf{i}+3\mathbf{k}$$

$$\underline{\underline{\mathbf{T}}} = 5\mathbf{ii}-\mathbf{ik}+3\mathbf{jk}+2\mathbf{ki}$$

$$\underline{\underline{\mathbf{S}}} = 2\mathbf{ii}+3\mathbf{ik}-2\mathbf{ki}$$

   evaluate:

   a. $\mathbf{a}\cdot\underline{\underline{\mathbf{T}}}$

   b. $\underline{\underline{\mathbf{T}}}\cdot\mathbf{a}$

   c. $\underline{\underline{\mathbf{S}}}\cdot\underline{\underline{\mathbf{T}}}$

   d. $\underline{\underline{\mathbf{S}}}:\underline{\underline{\mathbf{T}}}$

   e. $\mathbf{ab}$

7. Expressing vectors in terms of their scalar components and the three unit vectors:

$$\mathbf{v} = v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + v_3\mathbf{e}_3 = v_i\mathbf{e}_i$$

and expressing second-rank tensors in terms of their scalar components and the nine unit dyads:

$$\underline{\underline{\boldsymbol{\tau}}} = \tau_{11}\mathbf{e}_1\mathbf{e}_1 + \tau_{12}\mathbf{e}_1\mathbf{e}_2 + \ldots = \tau_{ij}\mathbf{e}_i\mathbf{e}_j$$

use the definition of dyadic product to deduce the formulae for computing the scalar components of the result of the following operations (you may assume the $\mathbf{e}_i$ constitute an orthonormal basis: i.e. $\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}$):

a. $\mathbf{v} \cdot \underline{\underline{\boldsymbol{\tau}}}$

b. $\underline{\underline{\boldsymbol{\tau}}} \cdot \mathbf{v}$

c. $\underline{\underline{\boldsymbol{\tau}}} : \underline{\underline{\boldsymbol{\sigma}}}$