Exam #1

(Closed Book, 110 minutes)

1.) Given the following:

$$\underline{\underline{\mathbf{T}}} = 5\mathbf{ii}-\mathbf{ik}+3\mathbf{jk}+2\mathbf{ki}$$
$$\underline{\underline{\mathbf{S}}} = 2\mathbf{ii}+3\mathbf{ik}-2\mathbf{ki}$$
$$\mathbf{v} = 4\mathbf{i}$$

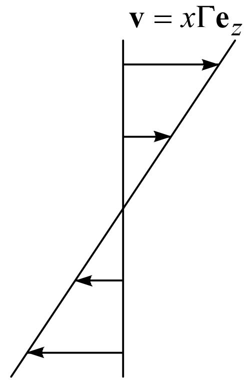
and

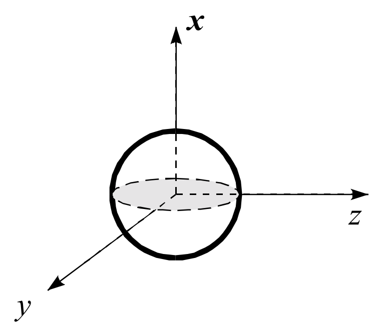
$$\underline{\underline{\mathbf{E}}} = (\underline{\underline{\mathbf{S}}}:\underline{\underline{\mathbf{I}}})\underline{\underline{\mathbf{T}}}$$

where $\underline{\underline{\mathbf{I}}}$ is the identity tensor. Evaluate the following:

10% a. E_{xx}
10% b. E_{xz}
10% c. $\underline{\underline{\mathbf{T}}} \cdot \mathbf{v}$

2.) A rigid sphere entrained in linear shear flow ($\mathbf{v} = x\Gamma\mathbf{e}_z$) will remain stationary (in this reference frame) but undergo rotation about the y-axis.





10% a. If the fluid can be treated as "ideal," does a velocity potential exist for this flow?

10% b. Can a solution be found using a stream function? You <u>don't need to find the stream function</u>, but explain how you reached your answer.

Hint: is flow axisymmetric? two dimensional? Some possibly useful relationships for spherical coordinates

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$
$$\mathbf{e}_x = (\sin\theta \cos\phi)\mathbf{e}_r + (\cos\theta \cos\phi)\mathbf{e}_\theta - (\sin\phi)\mathbf{e}_\phi$$
$$\mathbf{e}_y = (\sin\theta \sin\phi)\mathbf{e}_r + (\cos\theta \sin\phi)\mathbf{e}_\theta + (\cos\phi)\mathbf{e}_\phi$$
$$\mathbf{e}_z = (\cos\theta)\mathbf{e}_r - (\sin\theta)\mathbf{e}_\theta$$

10% c. What is the value of the angular rotation rate (a scalar) of the sphere in creeping flow?

Hint: <u>you don't need to solve for the velocity profile</u>. Consider the particle to be small enough to act like a "tracer" for the velocity profile.

5% d. What is the direction of this angular velocity (a vector)? You answer should be a unit vector (or linear combination of unit vectors) in either Cartesian or spherical coordinates. Be sure to get the sign correct.

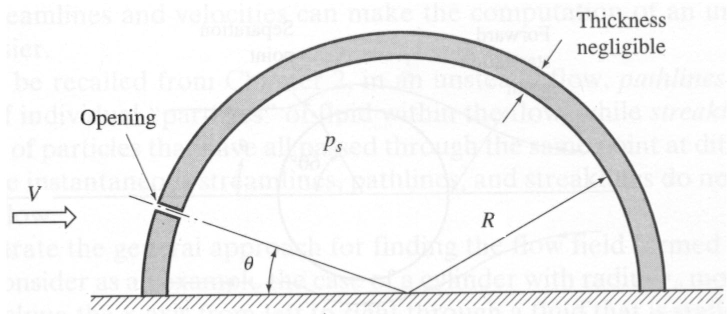
3.) Suppose the ideal fluid of the previous problem is replaced by a Newtonian fluid having viscosity μ. Evaluate the following quantities at $x = y = 0$ and $z \to \pm\infty$ (i.e. far from the disturbance caused by the sphere):

10% a. the rate of strain tensor (in Cartesian coordinates).

5% b. the viscous (deviatoric) stress tensor.

Be sure to indicate all nonzero components.

20% 4.) An arctic hut in the shape of a half-circular cylinder has a radius R. A wind of velocity V batters the hut and threatens to raise it off its foundations due to the lift of the wind. A clever occupant sizes up the situation and decides the net upward force on the hut can be eliminated by creating an opening in the wall, which will change the pressure p_s inside the hut. What is the angle θ such that the net force on the hut vanishes? Neglect the weight of the hut walls and also neglect any hydrostatic pressure variations. You may assume the size of the opening and the thickness of the hut wall are negligible compared to R.



Hint: The velocity and pressure profiles can be assumed to correspond to potential flow perpendicular to the axis of a long full-circular cylinder of the same radius, which are given by (θ in these equations is $\pi - \theta$ in the figure above):

$$v_r = V\left[1 - \left(\frac{R}{r}\right)^2\right]\cos\theta \qquad \text{and} \qquad v_\theta = -V\left[1 + \left(\frac{R}{r}\right)^2\right]\sin\theta$$

$$p(R,\theta) = p_0 + \tfrac{1}{2}\rho V^2\left(1 - 4\sin^2\theta\right)$$

In a related homework problem, the force exerted by potential flow around a half-circular cylinder (of radius R and length L) was found to be

$$\mathbf{F}_p = \tfrac{5}{3}\rho U^2 L R \mathbf{e}_y$$

You may use this result without re-deriving it.