1. Two spherical bubbles come into contact and fuse in a liquid of hydrostatic pressure $P$. The radius of the larger bubble is $R_a$ and that of the smaller bubble is $R_b$. There is a curved septum where the two bubbles are in contact. The radius of curvature of the septum is $R_s$. Derive an equation that relates $R_s$ to $R_a$ and $R_b$. (Note: the septum is a thin liquid film, so it has a vapor/liquid interface on both sides.) Why is the septum shaped the way it is?

![Diagram of two bubbles and a septum](attachment:diagram.png)

2. Show that the Thompson equation for nucleation of vapor bubbles in a boiling liquid is:

$$\Delta H_{\text{vap}} \left[ \frac{1}{T_0} - \frac{1}{T} \right] = R \ln \left( \frac{2\gamma + P_l}{P_l r} \right)$$

where $P_l$ is the pressure in the bulk liquid phase, set by the ambient pressure of the surrounding gas. If the heat of vaporization is 31 kJ/mol at 272 C and the surface tension of water is 18.4 mN/m at 272 C, find the radius of a vapor bubble in equilibrium with liquid at an ambient pressure of 1 atm and T=272 C.
3. NaCl particles (density=2.17 g/cm³) with a specific surface area of 4.25 x 10⁵ cm²/g show a supersaturation of 6.71% in ethanol (T=25 C). Assuming the crystals are uniform spheres, find their radius \( r \) and estimate the NaCl/ethanol interfacial tension \( \gamma \) from this solubility data.

Hint: for 1:1 salts in liquids, the Kelvin equation is:

\[
\frac{M \gamma}{\rho r} = RT \ln \left( \frac{S}{S_0} \right)
\]

where \( M \) is the molecular weight of the salt, \( \rho \) is the density of the salt, \( S \) is the solubility of the spherical particle and \( S_0 \) is the solubility of a flat particle.

4. The surface tension of isoamyl alcohol (liquid “B”) and its interfacial tension with water (liquid “A”) at T = 25 C are listed below. \( \gamma_B \) is the pure tension of pure B, while \( \gamma_{B(A)} \) represents the surface tension of A that is saturated with B, and \( \gamma_{AB} \) represents the A/B interfacial tension. The surface tension of water at 25 C is 72.2 mN/m.

a) Will isoamyl alcohol spread on water?
b) Will water spread on isoamyl alcohol?
c) Will either of the situations considered in parts (a) or (b) produce the “strange” behavior where a liquid initially spreads on the other, only to retract into a lens after the passage of time?

\[
\begin{align*}
\gamma_B &= 23.7 \text{ mN/m} \\
\gamma_{B(A)} &= 23.6 \text{ mN/m} \\
\gamma_{A(B)} &= 25.9 \text{ mN/m} \\
\gamma_{AB} &= 5 \text{ mN/m}
\end{align*}
\]

5. If \( \gamma_L \) and \( \theta \) are measured for a homologous series of liquids on a given low-energy solid surface, a plot of \( \cos \theta \) vs. \( \gamma_L \) results in a straight line. Estimate the critical surface tension for a Teflon surface with the following wetting characteristics. Is this value a good estimate of the surface energy (tension) of the Teflon?

<table>
<thead>
<tr>
<th>Compound</th>
<th>( \gamma_L ) (mN/m)</th>
<th>( \theta ) (deg)</th>
<th>Compound</th>
<th>( \gamma_L ) (mN/m)</th>
<th>( \theta ) (deg)</th>
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</thead>
<tbody>
<tr>
<td>hexadecane</td>
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<td>nonane</td>
<td>22.9</td>
<td>32</td>
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<td>hexane</td>
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<td>23.9</td>
<td>35</td>
<td>pentane</td>
<td>16.0</td>
<td>spreads</td>
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