Chemical Engineering Mathematics   06-262
Homework 4
Due: Thursday February 22, 2001

Matrix Algebra with MathCAD 2000

1. **(15 points)** Many of the problems that you tackled in the first three homework sets were time consuming by hand, but learning the method was vital. To practice using MathCAD and to see how much faster these problems are with a software package, repeat the problems listed below using MathCAD. Print out the results and turn them in with the rest of the homework. Include text as you would normally (i.e., number problems and explain what you are doing).
   a) Homework 1, problem 1
   b) Homework 1, problem 4
   c) Homework 2, problem 5
   d) Homework 3, problem 3

Exact Differential Equations

2. **(15 points)** The following differential equation is exact:

$$y' = \frac{y \cos(xy) - e^{2y}}{2xe^{2y} - x \cos(xy) + 2y}$$

   a) Show that the equation is exact.
   b) Write the general solution for the equation.
   c) For the condition that $y(x = \pi/2) = 0$ find the particular solution.

Integrating Factors

3. **(15 points)** The following two differential equations are not exact.

$$y' = \frac{-y(x + y + 1)}{x + 2y} \qquad \text{and} \qquad \frac{dy}{dx} = \frac{-6xy}{(4y + 9x^2)}$$

   a) Show that each equation is not exact.
   b) Find an appropriate integrating factor for each equation. If you cannot find a simple F(x) or F(y) integrating factor, then state that fact.
   c) Find the general solution for each equation.

Linear Differential Equations

4. (**15 points**) In class, I mentioned that linear first order differential equations are straightforward to solve. Nonlinear equations are much more difficult to deal with, but there is a special case where we can force a nonlinear equation to be linear. This is the case of the Bernoulli equation (see pg 36 of the textbook). Given the "almost" linear equation:

$$\frac{dy}{dx} + p(x)\, y = r(x)\, y^3$$

a) Substitute the function $u(x) = y(x)^{-2}$ and rewrite this equation as a first order differential equation in $u(x)$. Hint: it should be a linear equation.
b) Solve for $u(x)$.
c) Using the $u(x)$ from part (b), write the general solution for $y(x)$.

5. (**15 points**) Linear first order differential equations of the following form commonly appear in chemical engineering balance problems. Find the general and particular solutions for each. Consider the variables h_o, y_o, k, F, τ, α and ω to be constants.

$$y' + \frac{1}{\tau}\, y = 1 + \cos(\omega\, x) \quad \text{where } y(x = 0) = y_o$$

$$\frac{dh}{dt} = \alpha h + F \exp(-k\, t) \quad \text{where } h(t = 0) = h_o$$

Problem Solving with First Order Differential Equations

6. (**15 points**) A cylindrical tank of water ($\rho = 1000 \text{ kg/m}^3$) with cross sectional area, $A = 5 \text{ m}^2$, is draining through a valve in its base and being filled from the top at a flow rate given by $F_{in}(t)$. The flow rate of water through the valve is proportional to the height of water in the tank at any time. Initially, the tank is filled to a level of 10 m.

$F_{in}(t)$

Cross sectional area, A

h(t)
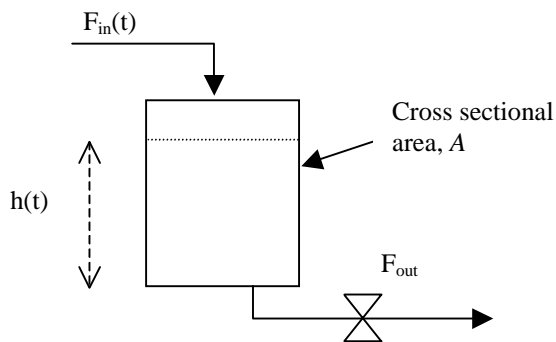
F_{out}

The outlet flow rate, F_{out}, through the valve is proportional to the height of water in the tank such that $F_{out} = h(t)/R$.

R is a constant where $R = 2 \text{ hr/m}^2$

a) State the conserved property, write the balance equation and cancel unnecessary terms. Keep the inlet flow rate in terms of $F_{in}(t)$, an unknown flow rate with units of m^3/hr.

b) Develop a differential equation for the height of water in the tank as a function of time.

c) For the case of no input flow ($F_{in}(t) = 0$), solve the separable FODE for the general and particular solutions. Plot h(t) versus time.

d) For the case of a constant input flow rate ($F_{in}(t) = 5\ m^3$/hr), solve the linear FODE for the general and particular solutions. Plot h(t) versus time.

e) For the case of an oscillating input flow rate $F_{in}(t) = F_o(1 + \sin(\omega t))$ where $F_o = 5\ m^3$/hr and $\omega = \pi/12\ hr^{-1}$, solve the linear FODE for the general and particular solutions. Plot h(t) versus time.