**Chemical Engineering Mathematics 06-262**
**Homework 3**
**Due: Thursday February 8, 2001**

Find all eigenvalues and eigenvectors by hand. Feel free to use a calculator or MathCAD to find the inverse of any matrices and roots of polynomials.

Eigenvalues and Eigenvectors

1) **(15 points)** For each of the following matrices, find the eigenvalues and eigenvectors of each. Write the eigenspace of each matrix.

a) $\begin{bmatrix} -2 & 0 & 3 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 4 \\ -2 & -4 & 2 \\ 2 & 4 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & -3 & 0 \\ -2 & 4 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}$

2) **(15 points)** In many cases, it is important to recognize matrices that will have imaginary rather than real eigenvalues. For the general 2x2 matrix:

$$\begin{bmatrix} a & c \\ d & b \end{bmatrix}$$

a) If a=b, what values of c and d will give imaginary rather than real eigenvalues?
b) If a=0, what values of b, c and d will give imaginary rather than real eigenvalues?
c) If a = c = 0, what values of b and d will give imaginary rather than real eigenvalues?

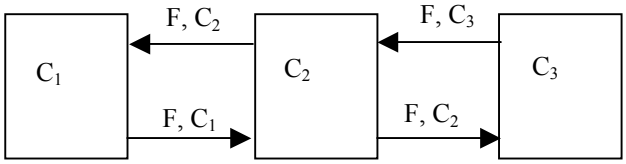
3) **(15 points)** Given the following 3x3 matrix A, consider how changes in the matrix cause changes in the eigenspace of the matrix.

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

a) Determine the eigenspace of the matrix **A**.
b) Determine the eigenspace of the matrix $\mathbf{A}^T$.
c) Determine the eigenspace of the matrix $\mathbf{A}^{-1}$.
d) Determine the eigenspace of the matrix formed by switching rows 2 and 3 of matrix **A**.
e) Comment on how each of these changes alters the eigenspace. Which matrix functions change the eigenvalues and how? Which change the eigenvectors and how?

4) **(15 points)** When we cover coupled differential equations later in the course, you will need to find the eigenvalues and eigenvectors of the coefficient matrix of a set of balance equations. To practice, set up the steady state balances on the following problem to develop a matrix expression $\mathbf{A\,c} = \mathbf{0}$. Then find the eigenvalues and eigenvectors of the coefficient matrix **A**. Solving for the steady state value of c is trivial and does not require the eigenvalues or eigenvectors of **A**, be sure that you know what the solution for **c** is without solving $\mathbf{Ac} = \mathbf{0}$.

Three mixers are connected as shown below. Initially, the first is filled with a high concentration of acid ($C_o$ = 0.5 kg/acid/kg total) and then the flow is started. The flow rates in all the streams are the same, F = 10 kg total/hr. The concentrations in each mixer, $C_i$, are in units of mass acid/mass total. Before setting up the balances, what is the steady state concentration in each of the three tanks (state your answer based on logic).



Differential Equations

5) **(15 points)** Solve the following separable differential equations. Find the general solution and then use the conditions given to find the particular solution.
   a) $\frac{df}{dx} = -3x$; $f(x=0) = f_o$
   b) $T' = \sin(\omega t) + 4$; $T(t=0) = 0$ and $\omega = \pi/4$
   c) $\frac{dc}{dt} + (t+1)c^3 = 0$; $c(t=0) = 1$
   d) $\gamma' = \frac{1}{t+1}$; $\gamma(t=0) = 1$
   e) $y' = y\, e^{2x} \sin(x)$ ; $y(x = \pi/2) = 1$

6) **(15 points)** A cylindrical tank is initially filled with water to a level of $h_o$. The tank has a constant cross section of A $m^2$. Water flows out of the tank through a valve at the bottom. The volumetric flow rate through the valve is proportional to the height of water in the tank, F = h/R where R is a resistance with units of $hr/m^2$.
   a) Develop an expression for the height of water in the tank as a function of time, h(t), in terms of the variables A, R and $h_o$.
   b) For a tank of A = 5 $m^2$ and resistance of 1 $hr/m^2$ which is initially filled to 10m, plot the height of water in the tank as a function of time.
   c) How long will it take for the tank to be half as full as when it started?