

Key to Homework #9

- 1.) (Geankoplis Prob. 10.4-1) This problem refers to Example 10.4-1 in which A diffuses through stagnant B. Then the overall mass transfer coefficient K_x' is related to the single-phase coefficients by (10.4-26):

$$\frac{1}{K_x} = \frac{(1-x_A)_{*M}}{K_x'} = \frac{(1-y_A)_{iM}}{m''k_y'} + \frac{(1-x_A)_{iM}}{k_x'} \quad (1)$$

Results obtained in Example 10.4-1:

$$k_y' = 1.465 \times 10^{-3} \text{ kgmol/s-m}^2$$

$$k_x' = 1.967 \times 10^{-3} \text{ kgmol/s-m}^2$$

$$(1-y_A)_{iM} = 0.709$$

$$(1-x_A)_{iM} = 0.820$$

In addition to this information, we need m'' and $(1-x_A)_{*M}$. These quantities are given by:

$$(10.4-17): \quad m'' = \frac{y_{AG} - y_{Ai}}{x_A^* - x_{Ai}} = \frac{0.380 - 0.197}{0.348 - 0.257} = 2.01$$

$$(10.4-27): \quad (1-x_A)_{*M} = \frac{(1-x_A) - (1-x_A^*)}{\ln \frac{1-x_A}{1-x_A^*}} = \frac{(1-0.100) - (1-0.348)}{\ln \frac{1-0.100}{1-0.348}} = 0.770$$

Substituting known values into (1):

$$\frac{1}{K_x} = \frac{0.770}{K_x'} = \frac{0.709}{\underbrace{(2.01)(1.465 \times 10^{-3})}_{240.8}} + \frac{0.820}{\underbrace{(1.967 \times 10^{-3})}_{416.9}} = 657.7$$

$$K_x = 1.519 \times 10^{-3} \frac{\text{kgmol}}{\text{s-m}^2 \text{ - mol frac}}$$

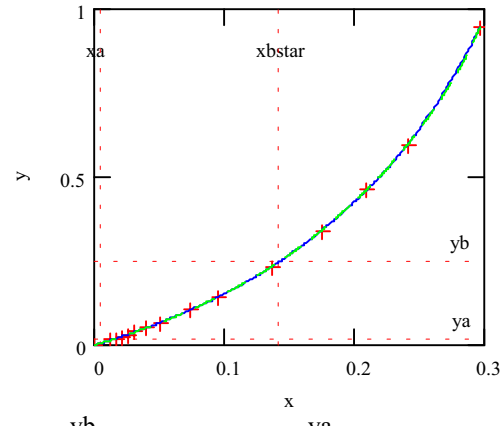
$$K_x' = 1.173 \times 10^{-3} \frac{\text{kgmol}}{\text{s-m}^2 \text{ - mol frac}}$$

(1) gives the total resistance as the sum of that for the gas and liquid phases. The percent resistance of the gas film is the gas film resistance divided by the total:

$$\% \text{ resistance} = \frac{240.8}{657.7} 100\% = 36.7\%$$

2.) (Geankoplis Prob. 10.6-4). The equilibrium curve for the solubility of ammonia in water at 1 atm, 30°C can be found on p886 of Geankoplis.

We have plotted the equilibrium in the figure at right. The red +’s are points from the table on p886. The overlapping blue and green curves are spline interpolations for $x^*(y)$ or $y^*(x)$.



Noting that the equilibrium curve curves upward, we tentatively assume that the minimum water flowrate can be determined by taking the outlet water to be saturated in ammonia. In other words, we assume that (x_b, y_b) lies on the equilibrium curve. For $y_b = 0.25$, this yields

$$x_b^* = 0.1429$$

Now we can calculate the water flowrate from an overall ammonia balance (10.6-3):

$$L'(X_b - X_a) = V'(Y_b - Y_a) \quad (2)$$

where V' is the ammonia-free air flowrate, which we can calculate from the given air inlet flowrate and composition:

$$V' = V_b(1 - y_b) = 181.4 \frac{\text{kgmol}}{\text{h}} (1 - 0.25) = 136.05 \frac{\text{kgmol air}}{\text{h}}$$

and where the X and Y are mole ratios, which are related to mole fractions by

$$X \equiv \frac{x}{1-x} \quad \text{and} \quad Y \equiv \frac{y}{1-y} \quad (3)$$

Thus

$$\begin{aligned} x_a = 0.005 &\rightarrow X_a = 0.005025 \\ x_b^* = 0.1429 &\rightarrow X_b^* = 0.1667 \\ y_a = 0.02 &\rightarrow Y_a = 0.0204 \\ y_b = 0.25 &\rightarrow Y_b = 0.3333 \end{aligned}$$

Substituting these values into (2) and solving for L' , we obtain the minimum water flowrate:

$$L'_{\min} = 263.3 \frac{\text{kgmol water}}{\text{h}}$$

We are told that the operating water flowrate is 1.5 times the minimum:

$$L' = 1.5L'_{\min} = 395 \frac{\text{kgmol water}}{\text{h}}$$

The operating line can be calculated from (10.6-4), which is the same as (2) except the balance is performed over a subset of the packed tower:

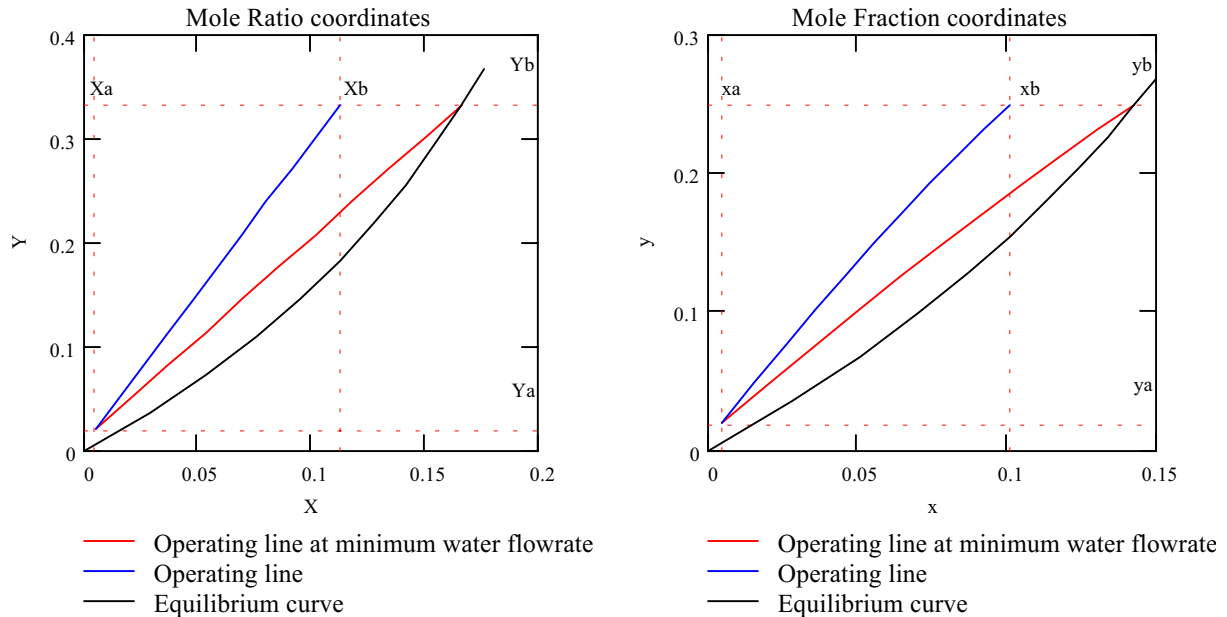
$$L'(X - X_a) = V'(Y - Y_a) \quad (4)$$

So we choose values of Y in the interval $Y_a < Y < Y_b$. For each Y , we calculate the corresponding X from (4). If desired, we can convert the mole ratios back into mole fractions by inverting (3):

$$x = \frac{X}{1 + X} \quad \text{and} \quad y = \frac{Y}{1 + Y} \quad (5)$$

This yields the following table

Y	y from (5)	X from (4)	x from (5)
0.0204 = Y_a	0.0200	0.0050	0.0050
0.0517	0.0492	0.0158	0.0156
0.0830	0.0766	0.0266	0.0259
0.1143	0.1026	0.0374	0.0360
0.1456	0.1271	0.0481	0.0459
0.1769	0.1503	0.0589	0.0556
0.2082	0.1723	0.0697	0.0652
0.2395	0.1932	0.0805	0.0745
0.2707	0.2131	0.0912	0.0836
0.3020	0.2320	0.1020	0.0926
0.3333 = Y_b	0.2500	0.1128	0.1014



3a.) (Geankoplis Prob. 10.6-6)

The following information is given in the problem statement:

$$y_2 = 0.04 \quad \text{and} \quad y_1 = 0.005$$

$$L_2 = L' = 68 \text{ kmol/hr} \quad \text{and} \quad x_2 = 0$$

$$V_1 = 57.8 \text{ kmol/hr} \quad \text{and} \quad D_T = 0.747 \text{ m}$$

$$k_y'a = 0.0739 \text{ kmol/(s-m}^3\text{)} \quad \text{and} \quad k_x'a = 0.169 \text{ kmol/(s-m}^3\text{)}$$

We are to calculate the tower height using $k_y'a$, in other words, using (10.6-18):

$$z = \left[\frac{V}{k_y'aS} \frac{(1-y)_{iM}}{1-y} \right]_{av} \int_{y_2}^{y_1} \frac{dy}{y-y_i}$$

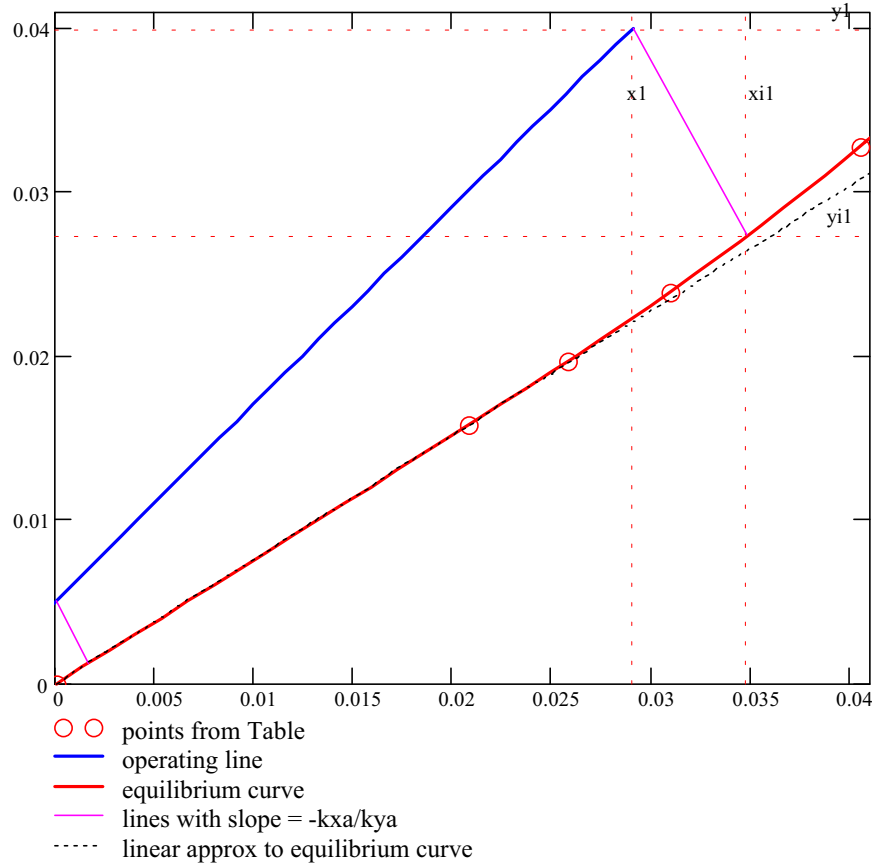
To evaluate the integral, we will need to evaluate y and y_i at several locations over the height of the packed tower. Generally $y(x)$ is given by the operating line; assuming one transferable component, the operating line is given by (10.6-3). First we evaluate

$$V' = V_1(1-y_1) = (57.8 \text{ kmol/hr})(1 - 0.005) = 57.5 \text{ kmol/hr}$$

Substituting this, along with $L' = 68 \text{ kmol/hr}$, $x_2 = 0$ and $y_2 = 0.04$, (10.6-3) becomes

$$\frac{y}{1-y} = 1.225 \frac{x}{1-x} + 0.005025 \quad (6)$$

This $y(x)$ or $x(y)$ is the operating line (actually a curve). The equilibrium curve $y^*(x)$ is obtained by a cubic spline interpolation of the data for 20°C in Appendix A.3-22 on p886. Both curves are plotted in the graph below along with the individual points (the circles) on the equilibrium curve taken from Appendix A.3-22.



In particular the operating line (6) can be used to obtain the ammonia concentration in the outlet water:

$$x_1 = x(y_1) = 0.0290$$

The point (x_1, y_1) is also shown on the figure above using dotted red lines. In this first part of the problem, we are to evaluate the height of packing using (10.6-18):

$$z = \underbrace{\left[\frac{V}{k'_y a S} \frac{(1-y)_{iM}}{1-y} \right]_{av}}_{H_y} \underbrace{\int_{y_2}^{y_1} \frac{dy}{y-y_i}}_{N_y} \quad (7)$$

Let's start by evaluating the integral. To do this we have to partition the interval of y values ($y_2 \leq y \leq y_1$) into a number of sub-intervals. For each bulk gas concentration y , we must determine the corresponding interfacial concentration y_i . To determine y_i , we note that the slope of a line connecting (x, y) with (x_i, y_i) has the value given by (10.6-30):

$$\frac{y - y_i}{x - x_i} = -\frac{k'_x a}{(1-x)_{iM}} \frac{(1-y)_{iM}}{k'_y a} = -2.287 \frac{(1-y)_{iM}}{(1-x)_{iM}} \quad (8)$$

where $(1-y)_{iM} = \frac{(1-y) - (1-y_i)}{\ln \frac{1-y}{1-y_i}}$ and $(1-x)_{iM} = \frac{(1-x) - (1-x_i)}{\ln \frac{1-x}{1-x_i}}$

Given a value for y , (8) is one equation involving three unknowns: x , x_i and y_i . The additional two equations arise from the constraints that

- 1) (x, y) must be a point on the operating line [or $x = x(y)$] and
- 2) (x_i, y_i) must lie on the equilibrium curve [or $x_i = x^*(y_i)$]

Substituting $x = x(y)$ and $x_i = x^*(y_i)$ into (8) yields one equation in one unknown (y_i):

$$\frac{y - y_i}{x(y) - x_i(y_i)} = -2.287 f(y, y_i)$$

This is a transcendental equation which must be solved numerically to yield $y_i(y)$: a perfect job for [Mathcad](#). Some numerical results are summarized in the table below:

y	x	y_i	x_i	$(y-y_i)/(x-x_i)$	$1/(y-y_i)$
0.0050	0.0000	0.0012	0.0016	-2.28	266
0.0085	0.0029	0.0037	0.0050	-2.28	210
0.0120	0.0058	0.0063	0.0083	-2.25	174
0.0155	0.0087	0.0088	0.0116	-2.26	149
0.0190	0.0116	0.0113	0.0150	-2.27	130
0.0225	0.0145	0.0139	0.0183	-2.27	116
0.0260	0.0174	0.0164	0.0216	-2.28	104
0.0295	0.0203	0.0190	0.0249	-2.28	95
0.0330	0.0232	0.0216	0.0282	-2.30	88
0.0365	0.0261	0.0244	0.0314	-2.29	82
0.0400	0.0290	0.0272	0.0346	-2.28	78

Notice how little the slope changes over this range of conditions. Using [Mathcad](#), it's easy to integrate the last column over y to obtain:

$$N_y = \int_{y_2}^{y_1} \frac{dy}{y - y_i} = 4.60$$

Let's compare this result with approximation (10.6-23) which should be quite accurate when the range of y -values are small enough. Taking the log-mean of $y_1 - y_{i1} = 0.0128$ and $y_2 - y_{i2} = 0.00376$, we obtain

$$(y-y_i)_M = 0.00739$$

Dividing this into the total change in gas-phase concentration gives an estimate of the number of transfer units according to (10.6-23):

$$N_y \approx \frac{y_1 - y_2}{(y - y_i)_M} = 4.74$$

This is quite close to the value of 4.60 obtained by numerical integration, but was obtained with much less work. To evaluate H_y from (7), we need to obtain average values (at bottom and top of column) of a number of quantities. These averages are simple arithmetic means:

$$V_1 = \frac{V'}{1 - y_1} = 57.8 \frac{\text{kmol}}{\text{hr}} \quad V_2 = \frac{V'}{1 - y_2} = 55.8 \frac{\text{kmol}}{\text{hr}} \quad V_{av} = \frac{V_1 + V_2}{2} = 56.8 \frac{\text{kmol}}{\text{hr}}$$

$(1-y)_{iM}$ is the logmean of $1-y$ and $1-y_i$. From the first line of the table above, $(1-y)_{iM}$ is calculated as 0.997; from the last line of the table $(1-y)_{iM}$ is calculated as 0.966. The mean of these two values is

$$(1-y)_{iM,av} = 0.982$$

$1-y_1$ is 0.96 whereas $1-y_2$ is 0.995; their mean is

$$(1-y)_{av} = 0.978 \quad (9)$$

The cross-sectional area of the tower is calculated from the given value of diameter:

$$S = \frac{\pi D_T^2}{4} = 0.438 \text{ m}^2$$

Finally we substitute into (7) to obtain:

$$H_y = 0.489 \text{ m}$$

and

$$z = H_y N_y = \mathbf{2.25 \text{ m}}$$

3b.) In this part, we are to re-compute the height using $K'_y a$, or (10.6-20):

$$z = \underbrace{\left[\frac{V}{K'_y a S} \frac{(1-y)_{*M}}{1-y} \right]_{av}}_{H_{Oy}} \underbrace{\int_{y_2}^{y_1} \frac{dy}{y - y^*}}_{N_{Oy}} \quad (10)$$

The overall mass transfer coefficient can be calculated from the single-phase coefficients using (10.4-15):

$$\frac{1}{K'_y a} = \frac{1}{k'_y a} + \frac{m'}{k'_x a} \quad (11)$$

The slope m' of the equilibrium curve is defined by (10.4-13):

$$m' = \frac{y_i - y^*}{x_i - x} \quad (12)$$

where (x, y^*) is a point on the equilibrium curve. At the top of the column, we have $x = x_2 = 0$ and $y_2^* = 0$. From our table above, we recall that $x_{i2} = 0.00165$ while $y_{i2} = 0.00124$. Using these values, we compute $m'_2 = 0.750$ from (12) and $K'_{y,a_2} = 0.0557 \text{ kmol}/(\text{s}\cdot\text{m}^3)$ from (11). Meanwhile at the bottom, we have $x = x_1 = 0.0290$ and $y_1^* = 0.0223$. From our table above, we recall that $x_{i1} = 0.0346$ while $y_{i1} = 0.0272$. Using these values, we compute $m'_1 = 0.867$ from (12) and $K'_{y,a_1} = 0.0536 \text{ kmol}/(\text{s}\cdot\text{m}^3)$ from (11). Averaging these two values gives

$$K'_{y,a_{av}} = 0.0546 \text{ kmol}/(\text{s}\cdot\text{m}^3).$$

$(1-y)^*_{M}$ is the logmean of $1-y$ and $1-y^*$. From the values in the previous paragraph, $(1-y)^*_{M}$ is calculated as 0.969 at the bottom of the column and 0.997 at the top. The mean of these two values is

$$(1-y)^*_{M,av} = 0.983$$

We already know $(1-y)_{av} = 0.978$ from (9). Substituting into (10), we obtain

$$H_{Oy} = 0.663 \text{ m}$$

The integral in (10) is evaluated as we explained in class. For a given bulk gas composition y , we can use the operating line (6) to deduce the corresponding liquid composition x . Given x , we can use the equilibrium curve to deduce y^* and the value of the integrand $1/(y-y^*)$. Plotting $1/(y-y^*)$ vs. y and integrating between y_2 and y_1 , we obtain (see [Mathcad](#)):

$$N_{Oy} = \int_{y_2}^{y_1} \frac{dy}{y - y^*} = 3.44$$

and

$$z = H_{Oy} N_{Oy} = \mathbf{2.28 \text{ m}}$$

4.) (Geankoplis Prob. 10.6-8) The following data is provided by the problem statement:

$$Z = 4.88 \text{ m} \quad \text{and} \quad D_T = 0.254 \text{ m} \quad \text{so} \quad S = \frac{\pi}{4} D_T^2 = 0.0507 \text{ m}^2$$

$$V' = 3.30 \frac{\text{kgmol}}{\text{hr}} \quad \text{and} \quad L' = 9.03 \frac{\text{kgmol}}{\text{hr}}$$

$$x_1 = 0.00363 \quad \text{and} \quad x_2 = 0$$

$$y_1 = 0.01053 \quad \text{and} \quad y_2 = 0.00072$$

The partial pressure acetone above aqueous solutions at 20°C is given in Table A.3-21 on page 886 of Geankoplis. The partial pressures can be converted into mole fractions by dividing by the total pressure. For example, a partial pressure of 30 mmHg yields

$$x = 0.0333: \quad y^* = \frac{30 \text{ mmHg}}{760 \text{ mmHg}} = 0.0394$$

Both y_1 and y_2 are smaller than this, so we can just linearly interpolate between the first two points in the table:

$$y^*(x) = \frac{0.0394}{0.0333}x = 1.182x$$

$$\text{Thus} \quad y_1^* = y^*(x_1) = 0.00423 \quad \text{and} \quad y_2^* = 0$$

Since all the mole fractions are so small, both the operating line and equilibrium curve will be linear. Then we can use (10.6-28) as the design equation. To use this equation, we will need the log-mean driving force

$$y_1 - y_1^* = 0.00630 \quad \text{and} \quad y_2 - y_2^* = 0.00072$$

The log-mean driving force can be evaluated from (10.6-25):

$$(y - y^*)_M = \frac{0.00630 - 0.00072}{\ln \frac{0.00630}{0.00072}} = 0.00257$$

Solving (10.6-28) for $K_y a$:

$$K_y a = \frac{\frac{V}{S}(y_1 - y_2)}{Z(y - y^*)_M} = \frac{3.30 \frac{\text{kgmol}}{\text{hr}} (0.01053 - 0.00072)}{(4.88 \text{ m})(0.00257)}$$

$$K_y a = 50.9 \frac{\text{kgmol}}{\text{m}^3 \cdot \text{hr}}$$

5a.) (Geankoplis Prob. 10.6-9) H_G is defined by (10.6-36):

$$H_G = \frac{V}{k'_y a S} \quad (13)$$

Instead of $k'_y a$, the problem statement gives us $k_G a$. The difference between these two is how the concentration is expressed. The driving force multiplying $k'_y a$ is in mole fraction units, whereas the driving force multiplying $k_G a$ is in partial pressure units. Partial

pressures can be converted into mole fraction by dividing by the total pressure, which is 1 atm in this problem; thus

$$k'_y a = k_G a P = 4.03 \frac{\text{lbmol}}{\text{ft}^3 \cdot \text{hr} \cdot \text{atm}} \times 1 \text{ atm} = 4.03 \frac{\text{lbmol}}{\text{ft}^3 \cdot \text{hr} \cdot \text{mol frac}}$$

The air flowrate is given in mass units, whereas we need molar flowrate in (13). The molecular weight of air is 29 lb_m/lbmol:

$$\frac{V}{S} = 95 \frac{\text{lb}_m}{\text{hr} \cdot \text{ft}^2} \times \frac{\text{lbmol}}{29 \text{ lb}_m} = 3.28 \frac{\text{lbmol}}{\text{hr} \cdot \text{ft}^2}$$

(13) yields:

$$H_G = \frac{3.28 \frac{\text{lbmol}}{\text{hr} \cdot \text{ft}^2}}{4.03 \frac{\text{lbmol}}{\text{ft}^3 \cdot \text{hr}}} = 0.813 \text{ ft}$$

The height of a liquid phase transfer coefficient is defined by (10.6-37):

$$H_L = \frac{L}{k'_x a S} \quad (14)$$

Instead of $k'_x a$, the problem statement gives us $k_L a$. The difference between these two once again is how the concentration is expressed. The driving force multiplying $k'_x a$ is in mole fraction units, whereas the driving force multiplying $k_L a$ is in lbmol/ft³. Molar concentrations of a solute are converted into mole fraction by dividing by the sum of the molar concentration of the solute plus the solvent. For very dilute solutions, we can just divide by the molar concentration of the solvent. For water, which has a density of 62.4 lb_m/ft³ and a molecular weight of 18 lb_m/lbmol:

$$C = \frac{62.4 \frac{\text{lb}_m}{\text{ft}^3}}{18 \frac{\text{lb}_m}{\text{lbmol}}} = 3.47 \frac{\text{lbmol}}{\text{ft}^3}$$

Then

$$k'_x a = k_L a C = \frac{16.6}{\text{hr}} \times 3.47 \frac{\text{lbmol}}{\text{ft}^3} = 57.5 \frac{\text{lbmol}}{\text{hr} \cdot \text{ft}^3}$$

Finally, the liquid flowrate must be converted into molar units by dividing by the molecular weight of water:

$$\frac{L}{S} = 987 \frac{\text{lb}_m}{\text{hr} \cdot \text{ft}^2} \times \frac{\text{lbmol}}{18 \text{ lb}_m} = 54.8 \frac{\text{lbmol}}{\text{hr} \cdot \text{ft}^2}$$

Substituting into (14):

$$H_L = \frac{L/S}{k'_x a} = \frac{54.8 \frac{\text{lbmol}}{\text{hr} \cdot \text{ft}^2}}{57.5 \frac{\text{lbmol}}{\text{hr} \cdot \text{ft}^3}} = 0.953 \text{ ft}$$

5b.) The height of an overall gas-transfer unit is defined by (10.6-38)

$$H_{OG} = \frac{V}{K'_y a S}$$

which is identical to H_{Oy} in the Notes. On page 137 of the 1998 Notes:

$$H_{Oy} = H_y + m \frac{V}{L} H_x \quad (15)$$

where H_x and H_y are identical to H_L and H_G , respectively. Now m is the slope of the equilibrium curve on mole fraction coordinates (i.e. y vs. x). We are given the equilibrium curve as

$$p_A = \frac{1}{1.37} c_A$$

where p_A is the partial pressure in atm and c_A is the concentration of acetone in the water expressed in lbmol/ft³. For very dilute solutions, c_A is just the mole fraction times the molar concentration of solvent (3.47 lbmol/ft³), whereas for 1 atm total pressure p_A is numerically equal to the mole fraction y . Thus the equilibrium curve is

$$y = \frac{3.47}{1.37} x = \underbrace{2.53}_m x$$

Substituting into (15):

$$H_{OG} = 0.813 \text{ ft} + 2.53 \underbrace{\frac{3.28 \frac{\text{lbmol}}{\text{hr} \cdot \text{ft}^2}}{54.8 \frac{\text{lbmol}}{\text{hr} \cdot \text{ft}^2}}}_{0.144 \text{ ft}} 0.953 \text{ ft} = 0.957 \text{ ft}$$

6.) (Geankoplis Prob. 10.6-11) The value of N_{OG} is the integral in (10.6-20), which simplifies for dilute solutions to an analog of (10.6-23):

$$N_{OG} = \int_{y_2}^{y_1} \frac{dy}{y - y^*} = \frac{y_1 - y_2}{(y - y^*)_M}$$

Substituting values of Prob. 10.6-8:

$$N_{OG} = \frac{y_1 - y_2}{(y - y^*)_M} = \frac{0.01053 - 0.00072}{0.00257} = 3.82$$

Since the total tower height is just $Z = N_{OG} H_{OG}$, we can calculate an experimental H_{OG} simply as:

$$H_{OG} = \frac{Z}{N_{OG}} = \frac{4.88 \text{ m}}{3.82} = 1.28 \text{ m}$$