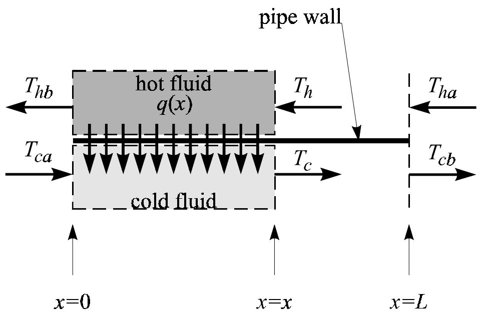
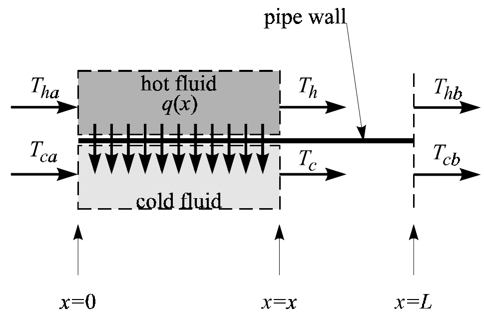
Key to Homework #1

1a.)



Countercurrent Flow



Co-Current Flow

For countercurrent flow, a energy balance over the portion of the heat exchanger from x=0 to x=x and either one of the fluids yields (2) from Lecture #3:

$$\begin{aligned} q &= \dot{m}_c c_{pc}\left(T_c - T_{ca}\right) \\ &= \dot{m}_h c_{ph}\left(T_h - T_{hb}\right) \end{aligned}$$

where q is the rate of heat transfer over this portion of the exchanger. For co-current flow, the analogous result is

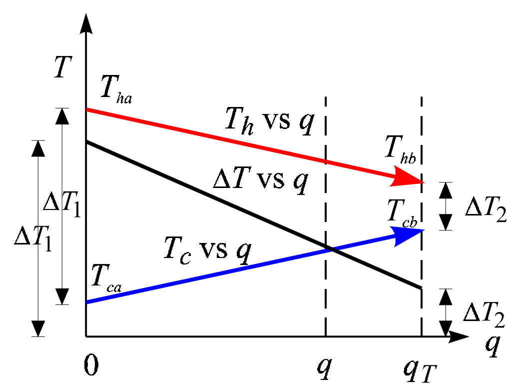
$$\begin{aligned} q &= \dot{m}_c c_{pc}\left(T_c - T_{ca}\right) \\ &= \dot{m}_h c_{ph}\left(T_{ha} - T_h\right) \end{aligned}$$

Solving for T_h and T_c, we still have a linear relationship with q:

$$T_c = T_{ca} + \left(\frac{1}{\dot{m}_c c_{pc}}\right) q$$

$$T_h = T_{ha} - \left(\frac{1}{\dot{m}_h c_{ph}}\right) q$$

although the relationship for T_h is different for co-current flow than for countercurrent. Thus ΔT vs q is still linear, so that



$$\frac{d(\Delta T)}{dq} = \text{const} = \frac{\Delta T_2 - \Delta T_1}{q_T - 0}$$

where $\Delta T_1 = T_{ha} - T_{ca}$

and $\Delta T_2 = T_{hb} - T_{cb}$

The remainder of the derivation is identical to that in Lecture #2. Eventually, we obtain

$$\boxed{q_T = UA_T \overline{\Delta T}_L} \quad \text{where} \quad \boxed{\overline{\Delta T}_L = \frac{\Delta T_2 - \Delta T_1}{\ln \dfrac{\Delta T_2}{\Delta T_1}}} \tag{1}$$

The only difference in the final formula are the values of the ΔT's:

$$\left.\begin{aligned} \Delta T_1 &= T_{ha} - T_{ca} \\ \Delta T_2 &= T_{hb} - T_{cb} \end{aligned}\right\} \text{for co-current flow}$$

whereas

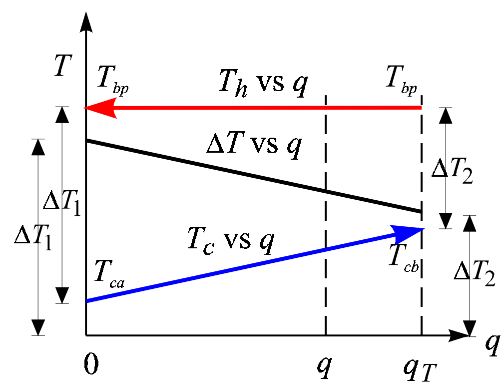
$$\left.\begin{aligned} \Delta T_1 &= T_{hb} - T_{ca} \\ \Delta T_2 &= T_{ha} - T_{cb} \end{aligned}\right\} \text{for countercurrent flow}$$

1b.) If hot stream is condensing steam, then its temperature remains constant as long as liquid and vapor co-exist:

$$T_h(x) = \text{const} = T_{bp}$$

for all $0 \le x \le L$. Once again, T_h, T_c and ΔT are all linear functions of q, so that the slope is a constant:

$$\frac{d(\Delta T)}{dq} = \text{const} = \frac{\Delta T_2 - \Delta T_1}{q_T - 0}$$



Again we eventually obtain the same design equation given by (1) except that the values of ΔT's are

$$\left.\begin{aligned} \Delta T_1 &= T_{bp} - T_{ca} \\ \Delta T_2 &= T_{bp} - T_{cb} \end{aligned}\right\} \text{for condensing steam}$$

2a.) <u>Geankoplis Prob. 4.3-8</u>

The design equation for heat exchangers is given by (4.3-13):

$$q = U_o\, A_o\, \Delta T = U_i\, A_i\, \Delta T \tag{2}$$

where the overall heat transfer coefficient U_o is computed from the sum-of-resistances formula (4.3-18):

$$\frac{1}{U_o} = \frac{1}{h_i}\frac{D_o}{D_i} + \frac{x_w}{k_w}\frac{D_o}{\overline{D}_L} + \frac{1}{h_o} \tag{3}$$

Geankoplis defines "resistance" as the driving force divided by the heat duty:

$$\sum R \equiv \frac{\Delta T}{q} = \frac{1}{U_o A_o} = \frac{1}{U_i A_i} \tag{4}$$

This represents the total resistance to heat transfer. The individual contributions can be determined by dividing (3) by $A_o = \pi D_o L$:

$$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{x_w}{k_w \overline{A}_L} + \frac{1}{h_o A_o} \tag{5}$$

We could also use (4.3-17), but we choose (somewhat arbitrarily) to use U_o instead. The problem statement gives us values for the two film coefficients:

$$h_i = 500 \text{ btu/(hr-ft}^2\text{-°F)}$$

$$h_o = 1500 \text{ btu/(hr-ft}^2\text{-°F)}$$

Knowing the pipe size (2", schedule 40), we can look up its dimensions using Table A.5-1. Using $L = 1$ ft, we can also compute the areas:

$$D_i = 2.067 \text{ in} \quad \text{and} \quad A_i = \pi D_i L = 0.541 \text{ ft}^2$$

$$D_o = 2.375 \text{ in} \quad \text{and} \quad A_o = \pi D_o L = 0.622 \text{ ft}^2$$

$$x_w = 0.154 \text{ in}$$

From this, we can compute D_L from (4.3-6):

$$\overline{D}_L = \frac{D_o - D_i}{\ln \dfrac{D_0}{D_i}} = 2.217 \text{ in} \quad \text{and} \quad A_L = \pi D_L L = 0.581 \text{ ft}^2$$

To calculate the overall transfer coefficient from (3), we will need the thermal conductivity of the pipe wall, which can be found in Table A.3-16. The value for 1% carbon steel does not seem to be very sensitive to temperature, so we will pick the value at 100°C:

$$k_w = 45 \text{ W/(m °K)}$$

Armed with these values, we can calculate each of the resistances in (5):

$$R_i \equiv \frac{1}{h_i A_i} = 7.01 \frac{\text{°K}}{\text{kW}} = 0.00370 \frac{\text{hr-°F}}{\text{btu}}$$

$$R_w \equiv \frac{x_w}{k_w \overline{A}_L} = 1.61 \frac{\text{°K}}{\text{kW}} = 0.0085 \frac{\text{hr-°F}}{\text{btu}}$$

$$R_o \equiv \frac{1}{h_o A_o} = 2.03\ \frac{°\text{K}}{\text{kW}} = 0.00107\ \frac{\text{hr-°F}}{\text{btu}}$$

Adding these resistances, we obtain

$$\sum R = 10.65\ \frac{°\text{K}}{\text{kW}} = 0.00562\ \frac{\text{hr-°F}}{\text{btu}}$$

Although it's not asked for, the relative contributions to the total resistance are:

inner film	pipe wall	outer film
0.658	0.151	0.191

Dividing the total resistance into the overall driving force ΔT = 220-70 = 150 °F

$$q = \frac{150\ °\text{F}}{0.00562\ \frac{\text{hr-°F}}{\text{btu}}} = 2.67 \times 10^4\ \frac{\text{btu}}{\text{hr}} = 7.82\ \text{kW}$$

2b.) The total resistance can also be expressed in terms of the U's by (4). Solving for U_i:

U_i = 329 btu/(hr-ft^2-°F) = 1.868 kW/(m^2-°C)

2c.) Solving (4) for U_o:

U_o = 286 btu/(hr-ft^2-°F) = 1.625 kW/(m^2-°C)

3a.) Geankoplis Prob. 4.5-3

The heat duty and outlet temperature for the cold water can calculated from (4.5-24):

$$mc_p \left(T_o - T_i\right) = m'c'_p \left(T'_i - T'_o\right) = q_T \qquad (6)$$

where the subscripts i and o denote inlet and outlet, respectively. The following info is given in the problem statement:

c'_p = 2.85 kJ/(kg-°K) m' = 7260 kg/hr T'_i = 377.6°K T'_o = 344.3°K

T_i = 288.8°K m = 4536 kg/hr U_o = 653 W/(m^2-°K)

The heat capacity of the cooling water can be read from Table A.2-5. Although the table shows heat capacity depends on temperature (and we don't know the outlet temperature), we note that heat capacity is relatively insensitive to temperature. So we just take some temperature above the inlet temperature and use the corresponding heat capacity:

c_p = 4.181 kJ/(kg-°K)

Using the available values, we can compute both q and T_o from (6):

$q = 191$ kW and $\mathbf{T_o = 325°K}$

For countercurrent flow, the inlet for the hot and cold fluids are on opposite ends of the heat exchanger, so the driving force at either end is

$$\Delta T_1 = T'_i - T_o = 52.5°\text{K}$$

$$\Delta T_2 = T'_o - T_i = 55.5°\text{K}$$

Computing the log-mean of these two values:

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = 54.0°K$$

Solving our design equation (2) for A_o:

$$A_o = \frac{q}{U_o \Delta T_{lm}} = \mathbf{5.43\ m^2}$$

3b.) For cocurrent flow, the inlet for the hot and cold fluids are on the same ends of the heat exchanger, so the driving force at either end is

$$\Delta T_1 = T'_i - T_i = 88.8°\text{K}$$

$$\Delta T_2 = T'_o - T_o = 19.2°\text{K}$$

Computing the log-mean of these two values:

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = 45.4°K$$

Solving our design equation (2) for A_o:

$$A_o = \frac{q}{U_o \Delta T_{lm}} = \mathbf{6.45\ m^2}$$

Notice that for cocurrent flow, the average driving force is smaller and so the area required is larger than for countercurrent flow. This is a general result.

4.)

(4.5-5)

A.3 Tables

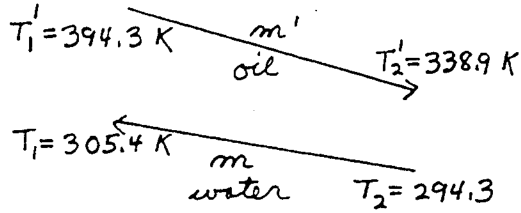
$c_{pm}(H_2O) = 4.183$ kJ/kg·K

m = kg water/h

c'_{pm} (oil) = 2.01 kJ/kg·K

$m' = 7258$ kg oil/h

$A_i = 5.11\ m^2$



Eq. (4.5-2)

$$q = m' c'_{pm} (T'_1 - T'_2)$$

$$= 7258(2.01)(394.3 - 338.9)$$

$$= 8.082 \times 10^5 \text{ kJ/h}$$

$$q = m c_{pm} (T_1 - T_2)$$

$$8.082 \times 10^5 = m(4.183)(305.4 - 294.3)$$

$$\boxed{m = 17\,420 \text{ kg } H_2O/h}$$

Eq. (4.5-27)

$$\Delta T_1 = 394.3 - 305.4 = 88.9 \text{ K}$$

$$\Delta T_2 = 338.9 - 294.3 = 44.6 \text{ K}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$$= \frac{88.9 - 44.6}{\ln(88.9/44.6)} = 64.22 \text{ K}$$

Eq (4.5-26)

$$q = U_i A_i \Delta T_{lm}$$

$$\frac{8.082 \times 10^5 (10^3)}{3600} \frac{J}{s} = U_i (5.11)(64.22)$$

$$\boxed{U_i = 686 \text{ W/m}^2 \cdot \text{K}}$$

5.) Geankoplis Prob. 4.5-7.

First we check to see if the flow is laminar or turbulent. For this we will need to calculate the Reynolds number. At 101.3 kPa and 283.2°K (close to the inlet conditions), the density and viscosity of air can be read from Table A.3-3:

101.3 kPa, 283.2°K: $\rho = 1.246\ \text{kg/m}^3$ (7)

$\mu = 1.78\times10^{-5}$ Pa-s

From the problem statement:

$$D_i = 12.7\ \text{mm} \quad \text{and} \quad v = 24.4\ \text{m/s}$$

The Reynolds number is calculated as

$$N_{\text{Re}} = \frac{\rho v D_i}{\mu} = 21{,}300$$

Since this is well above 2100, we know the flow will be turbulent. The Prandtl number turns out to be 0.7 and L/D for the tube is 120. Under these conditions, the heat transfer coefficient h_L can be calculated from (4.5-8). Unfortunately, using (4.5-8) in this trial-and-error problem requires extensive interpolation of the physical properties given in Table A.3-3, which depend on temperature. For air at 1 atm pressure, we can instead approximate the heat transfer coefficient h_L using (4.5-9):

$$h_L = \frac{3.52 v^{0.8}}{D^{0.2}} \tag{8}$$

where v is expressed in m/s and D is expressed in m, then h_L is obtained in W/m^2-°K. The heat-exchanger design equation is given by (4.5-26) and (4.5-27):

$$q = U_i A_i \Delta T_{lm} = U_i A_i \frac{\Delta T_2 - \Delta T_1}{\ln \dfrac{\Delta T_2}{\Delta T_1}} = h_L \pi D_i L \frac{(T_w - T_2) - (T_w - T_1)}{\ln \dfrac{T_w - T_2}{T_w - T_1}} \tag{9}$$

The problem statement suggests that we can treat the inside wall temperature T_w as a constant 372.1°K. This, in turn, suggests that the outside film resistance and the resistance of the pipe wall are negligible. Then (4.3-17) yields $U_i = h_i = h_L$, which was used to obtain the second equality above. The heat duty q is also related to the change in temperature of the air stream (4.5-24):

$$q = m c_p (T_2 - T_1) \tag{10}$$

Equations (9) and (10) constitute two equations in two unknowns: the heat duty q and outlet temperature T_2. Eliminating q leaves a trancendental equation in T_2 which will have to be solved numerically. Before proceeding, we need to evaluate some of the symbols appearing in these two equations. The mass flowrate is calculated as $\rho v A_c$. Although the fluid density ρ and cross-sectional average velocity v inside the tube depend on temperature, the mass flowrate does not. So we can use any temperature to evaluate m. Since the problem statement provides a value for v at the inlet conditions, we will evaluate ρ at those some conditions. Correcting (7) for temperature using the ideal gas law:

$$\rho = \left(1.246 \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{283.2°\text{K}}{T}\right)$$

At T = 288.8°K, this yields $\rho = 1.222\ \text{kg/m}^3$

Substituting D_i = 12.7 mm and L = 1.52 m, the heat exchanger area is

$$A = \pi D_i L = 0.0606\ \text{m}^3$$

while the cross-sectional area of the pipe is

$$A_c = \frac{\pi D_i^2}{4} = 1.267 \times 10^{-4}\ \text{m}^2$$

Using v = 24.4 m/s at the inlet conditions, the mass flowrate is

$$m = \rho v A_c = 3.78\times10^{-3}\ \text{kg/s} \tag{11}$$

The heat capacity of air is virtually insensitive to temperature (see Table A.3-3):

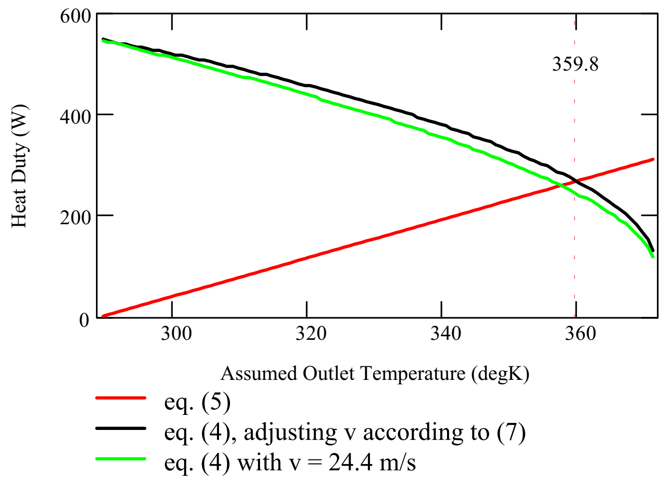
$$c_p = 1.009\ \text{kJ/kg-°K}$$

Finally, in order to evaluate h_L from (8) we will need to correct the average fluid velocity v for temperature. In order to obtain the same mass flowrate from (11) at each temperature, v must be inversely proportional to ρ or directly proportional to T:

$$v = \left(24.4 \frac{\text{m}}{\text{s}}\right)\left(\frac{T}{288.8\ °\text{K}}\right) = \left(24.4 \frac{\text{m}}{\text{s}}\right)\left(\frac{T_1 + T_2}{2 \times T_1}\right) \tag{12}$$

To obtain the average h_L we should use an average v which we evaluate at the average of the inlet and outlet air temperatures: $(T_1 + T_2)/2$, where T_1 = 288.8 °K is the inlet air temperature.

Now we are in a position to evaluate the heat duty q from either (9) or (10), assuming different values for the outlet temperature T_2. The inlet temperature remains fixed at T_1 = 288.8 °K. The results are summarized in the graph at right. The outlet temperature ranges from the inlet value 288.8 °K to the wall temperature 372.1 °K. The actual outlet temperature corresponds to the intersection of these two curves:



$$T_2 = 359.8 \text{ °K}$$

which corresponds to an average heat transfer coefficient of

$$h_L = 119.1 \text{ W-m}^{-2}\text{-°K}^{-1}$$