Is mathematics obsolete?

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Mathematics vs. science

Philosophers have long recognized a distinction between mathematical and scientific reasoning.

Mathematics	**Science**
about abstract objects	about the world
neither temporal nor spatial	object in time and space
rational	empirical
deductive	inductive
from axioms to conclusions	from data to laws
certain	fallible
exact, precise	approximate

Mathematics vs. science

This has played out in philosophy in various ways.

Mathematics	**Science**
Platonic forms	Aristotelian forms
learned by reflection	learned from experience
relations of ideas	matters of fact
a priori	a posteriori
analytic	synthetic

Mathematics vs. science

Philosophical differences span multiple axes:

- ontological: the nature of the objects involved
- generative: how we come to have the knowledge
- epistemological: the proper means of justification
- personal preference

Mathematics vs. science

The differences are tracked by distinct approaches to AI:

Mathematics	**Science**
symbolic methods	neural methods
automated reasoning	machine learning
formal methods	data science

Modern AI strongly favors the right-hand side.

Given recent successes of neural methods, those of us who work with symbolic methods are uneasy.

Mathematics vs. science

"There are two quite different paradigms for AI. Put simply, the logic-inspired paradigm views sequential reasoning as the essence of intelligence and aims to implement reasoning in computers using hand-designed rules of inference that operate on hand-designed symbolic expressions that formalize knowledge. The brain-inspired paradigm views learning representations from data as the essence of intelligence and aims to implement learning by hand-designing or evolving rules for modifying the connection strengths in simulated networks of artificial neurons."

(Yoshua Bengio, Yann Lecun, and Geoffrey Hinton, 2018 Turing Award winners)

Mathematics vs. science

Outline

- Mathematical knowledge vs. scientific knowledge
- Formal methods in mathematics
- Machine learning in mathematics
- Is mathematics obsolete?
- (spoiler alert:) Why it isn't

Formal methods in mathematics

Since the early twentieth century, we have known that mathematics can be represented in formal axiomatic systems.

Computational "proof assistants" allow us to write mathematical definitions, theorems, and proofs in such a way that they can be

- processed,
- verified,
- shared, and
- searched

by mechanical means.

Formal methods in mathematics

A small revolution is underway.

Growing numbers of young mathematicians are contributing to the development of online repositories.

There have been a number of striking landmarks and success stories.

There have been articles in Nature, Quanta, and so on.

You can find talks on YouTube.

Formal methods in mathematics

MATHEMATICS AND THE FORMAL TURN

JEREMY AVIGAD

Abstract. Since the early twentieth century, it has been understood that mathematical definitions and proofs can be represented in formal systems systems with precise grammars and rules of use. Building on such foundations, computational proof assistants now make it possible to encode mathematical knowledge in digital form. This article enumerates some of the ways that these and related technologies can help us do mathematics.

Introduction

One of the most striking contributions of modern logic is its demonstration that mathematical definitions and proofs can be represented in formal axiomatic systems. Among the earliest were Zermelo's axiomatization of set theory, which was introduced in 1908, and the system of ramified type theory, which was presented by Russell and Whitehead in the first volume of *Principia Mathematica* in 1911. These were so successful that Kurt Gödel began his famous 1931 paper on the incompleteness theorems with the observation that "in them all methods of proof used today in mathematics are formalized, that is, reduced to a few axioms and rules of inference." Cast in this light, Gödel's results are unnerving: no matter what mathematical methods we subscribe to now or at any point in the future, there will always be mathematical questions, even ones about the integers, that cannot be settled on that basis—unless the methods are in fact inconsistent. But the positive

Formal methods in mathematics

Executive summary: formal methods can be useful for

- verifying theorems
- correcting mistakes
- gaining insight
- building libraries
- searching for definitions and theorems
- refactoring proofs
- refactoring libraries
- engineering concepts
- communicating
- collaborating
- managing complexity
- managing the literature
- teaching
- improving access
- using mathematical computation
- using automated reasoning
- using AI

The technology holds a lot of promise.

Machine learning methods in mathematics

Mathematicians have begun using machine-learning techniques to discover new mathematics.

See talks by Marc Lackenby, Adam Wagner, and Geordie Williamson at the recent meeting, *Machine Assisted Proof*, and the Institute for Pure and Applied Mathematics.

But:

- These applications are very specialized, targeted, and focused.
- All of them were used to generate insight for ordinary, pen-and-paper proofs.

Machine learning methods in mathematics

There have been a number of recent papers on the use of machine learning to prove theorems automatically.

But:

- Successes and impact on practice has been limited so far.
- Symbolic representations play a key role.

So what am I worried about?

Is mathematics obsolete?

Over and over again, we have seen ML and big data take over domains where symbolic methods were dominant:

- expert systems to big data in decision making
- symbolic models to connectionist models in cognitive science
- heuristic search to AlphaZero in game playing
- symbolic models to statistical models and LLMs in natural language processing
- explicit algorithms to neural methods in computer vision

Is mathematics obsolete?

Carnegie Mellon was in the heart of the symbolic AI revolution in the twentieth century.

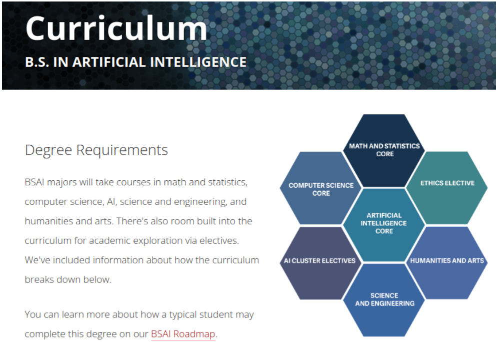
Herbert Simon helped found the Graduate School of Industrial Administration, the School of Computer Science, the Department of Psychology, and the Department of Philosophy.

He was a Nobel Prize winner and a Turing Award winner, and a pioneer in AI and cognitive science.

In the early 1950s, he developed the *Logic Theorist* with Allen Newell and Cliff Shaw.

Is mathematics obsolete?

A few years ago, Carnegie Mellon established an undergraduate major in artificial intelligence. The web pages include an overview, degree requirements, and a sample curriculum.



The word “logic” does not appear.

Is mathematics obsolete?

From the New York Times

What is artificial intelligence?

Let's start at the beginning.

The term "artificial intelligence" gets tossed around a lot to describe robots, self-driving cars, facial recognition technology and almost anything else that seems vaguely futuristic.

A group of academics coined the term in the late 1950s as they set out to build a machine that could do anything the human brain could do — skills like reasoning, problem-solving, learning new tasks and communicating using natural language.

Progress was relatively slow until around 2012, when a single idea shifted the entire field.

It was called a **neural network**. That may sound like a computerized brain, but, really, it's a mathematical system that learns skills by finding statistical patterns in enormous amounts of data. By analyzing thousands of cat photos, for instance, it can learn to recognize a cat. Neural networks enable Siri and Alexa to understand what you're saying, identify people and objects in Google Photos and instantly translate dozens of languages.

Is mathematics obsolete?

"[The] symbolic AI program aimed at achieving system 2 abilities, such as reasoning, being able to factorize knowledge into pieces which can easily be recombined in a sequence of computational steps, and being able to manipulate abstract variables, types, and instances. We would like to design neural networks which can do all these things while working with real-valued vectors so as to preserve the strengths of deep learning which include efficient large-scale learning using differentiable computation and gradient based adaptation, grounding of high-level concepts in low-level perception and action, handling uncertain data, and using distributed representations."

(Bengio, Lecun, and Hinton)

Is mathematics obsolete?

To some extent, I am conflating "mathematical reasoning" with "symbolic reasoning."

But I take it that one of the hallmarks of mathematical reasoning is that it employs symbolic reasoning with precise rules and concepts.

Modern AI threatens to replace that.

Is mathematics obsolete?

There is a strong aesthetic component to mathematics.

- We do mathematics because it is useful.
- We do mathematics because it is beautiful, and because we enjoy it.

But part of the aesthetic is that mathematics, and precise, rigorous thought, is broadly useful.

And maybe modern technology will change that.

Is mathematics obsolete?

For those of us who use symbolic methods, the feeling of anxiety is visceral.

It's not envy: we have great respect for people who work in machine learning.

It's just not what we do.

Why we need symbolic methods in AI

Perhaps neural networks will have to incorporate or interact with symbolic algorithms to do mathematics.

But that just pushes the question back to why we want to do mathematics.

Why we need mathematics

Many people have pointed out that, when asked factual questions, large language models often get the answers wrong.

But what will happen when they start to get the answers right?

That just needs better databases and database access, not substantial mathematical reasoning.

Why we need mathematics

Mathematics plays a central role in engineering and science.

But it is conceivable that AI will be able to design bridges, synthesize chemicals, and forecast the stock market without explicit mathematical models.

Why we need mathematics

Some have argued that machine learning systems need symbolic components so that they can *explain* what they are doing to us.

We need to *understand* what they are doing so that we can make sure that they are *aligned* with our goals.

This is compelling, but I think the crux of the matter goes beyond extracting explanations.

Why we need mathematics

Every month, my bank sends me a statement telling me how much money I have in my account.

It uses a simple symbolic algorithm.

Nobody thinks we should use a neural network for that. Why not?

Why we need mathematics

We use dollar amounts to make decisions, individually and collectively.

They form the basis for individual deliberation and planning.

Mathematical precision is also important for collaboration, which is deliberation and planning on a collective level.

Characterizing mathematics

Some things that characterize mathematics:

- The use of abstraction.
- The goal of solving hard problems.
- The use of rigor and precision.

The last component is essential:

- Picasso was really good at abstraction.
- Securing peace in the Middle East is a hard problem.

Characterizing mathematics

Mathematics is about having big ideas, deep insights, and far reaching intuitions *about things that can be made precise*.

What moves us are the power of the ideas, insights, and intuitions.

But the precision allows us to:

- communicate with one another,
- come to stable consensus as to whether an argument is correct,
- and carry out complex reasoning efficiently.

This is what symbolic methods support.

Characterizing mathematics

These skills are generally useful for

- formulating goals
- planning
- deliberating

both individually and collectively.

These are part of what it means to be rational, and to be human.

Why we need mathematics

Philosophers have used similar strategies to explain what makes mathematical knowledge certain.

- It's stamped on our souls.
- It's part of our cognition.
- It's part of what we mean by reasoning.
- It's part of our shared linguistic norms.

We're looking for an analysis of what makes mathematical reasoning valuable.

Why we need mathematics

An essential part of being human is being *rational*.

Being *rational* means having goals, and

- deliberating,
- planning,
- discussing,
- debating,
- and coordinating with others

to attain them.

Having a goal means having something that can be expressed, recorded, reasoned about, and communicated.

This means that they are inherently symbolic, and linguistic.

Why we need mathematics

I am not saying mathematicians are supremely rational!

Compare to physical ability:

- Some professions require high degrees of strength, speed, physical skill.
- We admire it in athletes.
- In everyday life, these attributes are generally useful.

Not everyone needs to be a rocket scientist or a Fields medalist, but the ability to reason and communicate precisely is generally useful, and it's important to us.

Summary

Remember the initial distinctions:

Mathematics	**Science**
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The central question: why do we need mathematical reasoning?

Summary

As long as we have goals, and as long as we need to deliberate by ourselves or with others, we will need precise concepts and precise communicative and inferential norms.

Neural methods can help us gather and process the data, but symbolic methods will help us reason about it.

Mathematicians can relax. We will always need both of these.

For all the anxiety, I am generally optimistic about the role of technology.

Reflecting on what is important to us and what we want technology to do will help us use it wisely.