Some answers to ten questions about the history of mathematics raised by young people

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This document provides responses to a list of questions compiled by Lizhen Ji and Chang Wang, culled from a longer list of 181 questions raised by young historians in China.

It's a pleasure to see such enthusiasm for the history of mathematics among young scholars, and I am happy to try to answer the questions, with the following caveat: *I am not a historian.* Both my undergraduate and graduate degrees are in mathematics, although I have long had an interest in the history and philosophy of the subject. My PhD is in a branch of mathematical logic known as *proof theory*, which focuses on the grammar, rules, and linguistic norms and expectations of mathematical practice. That perspective is fundamental to almost everything I do as a researcher.

One of the great pleasures of my career is that I have been able to devote time to the study of the history of mathematics, and that has had profound effects on how I think about the subject. In the 2003-2004 academic year, I was supported by a grant from the Andrew W. Mellon Foundation. This gave me the opportunity to the development of the theory of ideals in the nineteenth century, and to talk about the history of mathematics with Harold Edwards, Jeremy Gray, and John Stillwell. Work I did with John Mumma and Edward Dean around 2007 had me spending a lot of time reading the first four books of Euclid, and around 2012, I began a deep study of the history of Dirichlet's theorem on primes in an arithmetic progression with Rebecca Morris. At Carnegie Mellon, I have also had the pleasure of teaching graduate seminars (some with Ken Manders) and undergraduate seminars on the history of mathematics.

There are lots of ways of coming at the subject. One can focus on the social, cultural, and political factors that have shaped the evolution of mathematical practice, or one can focus on the real-time production of mathematics, relying on detailed archival study of letters and notes to illuminate the process by which mathematical ideas germinate and mature to the point where they are ready to enter the mathematical literature. I have done neither. Rather, I have studied the historical mathematical literature itself, and I have thought about the way mathematics has changed over time. In short, I have approached the history of mathematics as a history of ideas. This is not a value judgement. It is just a reflection of my personal preference, and what my personal circumstances have allowed.

It is fair to say that my primary interest has not been in history itself, but on what history has to tell us about what it means to do mathematics. I have adopted a strategy I learned from Ken Manders: find an episode in the history of mathematics that is generally recognized as something important, something like the algebraization of geometry or the invention of calculus (each of these is what Ken would call "a big deal difference"), and then try to say exactly what changed. Implicit in notion of mathematical progress is the understanding that there is a difference between the epistemic state of mathematics before the development and the one after. Trying to spell out exactly what changed tells us something important about the subject, its goals, and the forces that drive it.

I have always focused on the mathematical texts. This probably stems from my training as a proof theorist, though it also probably stems from the orthodox Jewish talmudic tradition that I was exposed to in elementary school and high school. When we try to make sense of mathematics, what we have direct access to is the mathematical literature, and there is a lot of it. I have learned that when you read a mathematical text carefully with the big questions in mind, you always notice things. What does the text say, and how does it say it? How is the manner of expression different from what has come before? What does the author make explicit, and what does the author leave out? What does the author assume of the reader, and what assumptions and expectations must the reader have in order to make sense of it? When I teach seminars to undergraduates, the project assignments are always the same: pick an interesting piece of mathematics, read it carefully, and tell me what you see. To paraphrase Yogi Berra, you can see a lot by looking.

All this context is important for making sense of the answers below. For the original questions, please refer to the document by Ji and Wang.

1. There are lots of different approaches to the history of mathematics. The most important thing is to choose a topic you find interesting, and to choose a method of analysis that you are comfortable with. Both of those will depend on your background and training.

   For me, the driving questions are: how does mathematics work? How does mathematics help us solve hard problems and think more efficiently? How do mathematical abstractions help us get a better grip on complex phenomena? It's probably best not to come to the history of mathematics with overly specific problems or questions. If you come to history with a vague sense of what you are looking for, you'll know it when you find it.

2. It's hard to say what counts as new research. When you focus on published literature, as I do, you are looking at things are many others have looked at before. But the wonderful thing about the subtlety and complexity of mathematics is that there are always new things to notice. Often apparently small things—like the form of a definition or an offhand remark before a theorem—are really interesting when interpreted in the right context.

3. Let's recognize that different audiences can learn different things from the history of mathematics. A working number theorist can learn something interesting from technical developments in their field, whereas artists and politicians can learn something interesting about the cultural and practical influences that shape mathematics. It's o.k. if historical work addresses different audiences, but for a particular work is to be successful, the author has to keep the audience in mind.

4. The history of mathematics is far too rich and complicated for any one approach to do it justice, so multiple perspectives are essential. Collaboration is also essential: complementary approaches often cast new light on your own approach, and, conversely, having to explain your work to someone who does not think about things the same way you do is an important way of coming to terms with what is genuinely interesting and important.

5. As a mathematician by training, the way I understand any historical document is to translate it into contemporary language and relate it to the mathematics that I know. I always feel a bit guilty about that, because I know that one *should* try to understand the document in its historical context. But to some extent, that's impossible: we can neither forget the

mathematics we know nor duplicate the training and experiences of a mathematician of the past. The best we can do is to be sensitive to the issues. Having understood a piece of mathematics in our own terms, we can think about how the presentation differs from modern ones, and that should help us come to terms with the ways that the historical understanding differs from our own.

6. I think most of the important qualities of a good historian are obvious: the ability to read carefully, critically, and thoughtfully, the ability to synthesize and analyze ideas, and the ability to communicate well. Maybe the most important quality of a good historian is to love mathematics and its history. It's hard not to get caught up in an author's enthusiasm.

7. I have only published one paper in a history journal. But every discipline has its norms and expectations, and it helps to be open to criticism. Talk to as many people as you can who are working in the field, listen to what they say, and when a paper is rejected, learn as much as you can from the feedback and criticism.

8. See the narrative above as to how and why I came to study the history of mathematics.

9. Read the originals. It helps to have a good guide. I still learn things from Struik's sourcebook on the history of mathematics, and Stillwell's explanatory preface to his translation of Dedekind's theory of algebraic integers was a great help. I actually learned Galois theory from Harold Edwards' wonderful historical presentation, and that has helped me understand the modern viewpoint.

10. Beyond the comments above, I only want to underscore the pleasure and the importance of studying the history of mathematics. Understanding mathematical thought and how we got here is richly rewarding, and it is an important part of understanding what it means to be human.